Double slit. $E_{inc}(x,y) = E_0$ $A = \text{rect}\left(\frac{x-d}{2a}\right) + \text{rect}\left(\frac{x+d}{2a}\right)$

drop prefactors one-dim.

$$E_{dif} \propto \mathcal{F}\left\{ \text{rect}\left(\frac{x-d}{2a}\right) + \text{rect}\left(\frac{x+d}{2a}\right) \right\}$$

just like double pulse

$$E_{dif} \propto a\left(e^{i\beta_x d}\,\text{sinc}(\beta_x a) + e^{-i\beta_x d}\,\text{sinc}(\beta_x a)\right)$$

$$= 2\,\underbrace{\text{sinc}(\beta_x a)}_{\text{broad envelope}}\;\underbrace{\cos(\beta_x d)}_{\text{interference fringes.}}$$

Or $A(x) = \text{rect}\left(\frac{x}{2a}\right) \otimes \left(\delta(x-d) + \delta(x+d)\right)$

$$\mathcal{F}\{A(x)\} = \mathcal{F}\left\{\text{rect}\left(\frac{x}{2a}\right)\right\} \cdot \mathcal{F}\{\delta(\;) + \delta(\;)\}$$

Non-normal incidence

same $A(x)$ $E_{inc} \sim e^{i(k_z z + k_x x)}$

$k_z = k_0 \cos\alpha$

$k_x = k_0 \sin\alpha$

evaluate at $z=0$

$E_{inc} \sim e^{ik_0 x \sin\alpha}$

Now output has a shift

$$E_{inf} \propto \mathcal{F}\left\{ e^{ik_0 \sin\alpha \cdot x} A(x) \right\}$$

$$\beta_x' \to \beta_x + k_0 \sin\alpha$$

since $\beta_x \equiv -k_0 \sin\theta_x$ $\beta_x' = k_0(\sin\alpha - \sin\theta_x)$

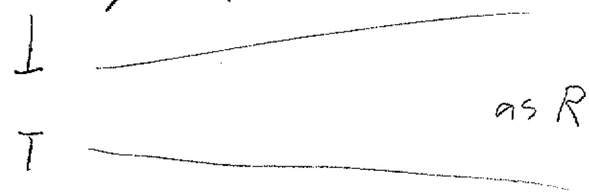
$= k_0(\sin\alpha - X/R)$

max at $\beta' = 0$ or $X = R\sin\alpha$

Far-field pattern at focal plane of lens:

- Fourier-transforming properties of lens

Consider single slit:

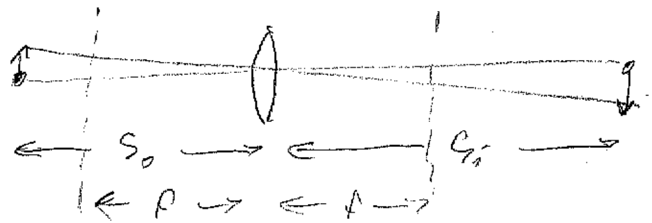


as $R \to \infty$ see $E \propto \text{sinc}\left(\frac{k_0 X a}{R}\right)$

like an image at ∞

basic lens review:

$1/s_o + 1/s_i = 1/f$



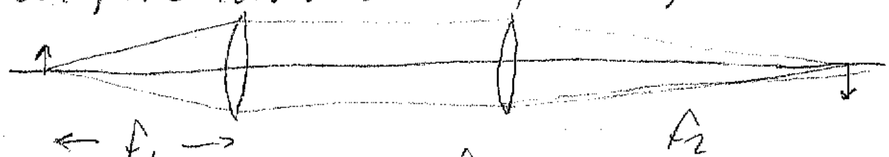
magnification $M = -\frac{s_i}{s_o}$

move $s_o \to \infty$ (to left) $s_i \to f$

s_o at $2f$, $s_i = 2f$, $M = -1$

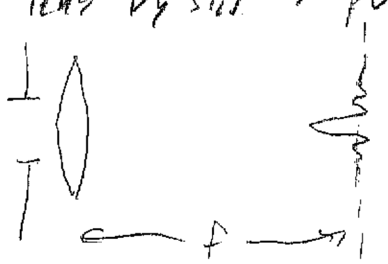
move $s_o \to f$ $s_i \to \infty$ (to right)

- here, can place second lens to put image at f_2



$M = -f_2/f_1$

$\therefore$ since Fraunhofer pattern is the diffn pattern at $R \to \infty$,
place lens by slit $\to$ puts Fraunhofer pattern at focus.



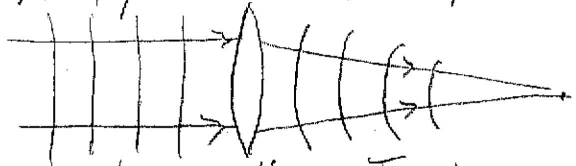
Fourier transform at lens f: derivation:

Start with Fresnel integral:

$$E_{diff}(X,Y) = \frac{e^{ikR}}{i\lambda R}\iint E_{in}(x,y)\, e^{ik\left(\frac{x^2+y^2}{2R}\right)} e^{-ik\left(\frac{xX}{R}+\frac{yY}{R}\right)} dx\,dy$$

we will add effect of lens at input

lens takes plane wave → spherical wave



rays are local $\vec{k}$

- always ⊥ wavefront.

can prove by writing $T(\rho) = e^{i\phi(\rho)}$ where $\phi(\rho)$ accounts for phase shift from lens thickness.

- thicker center retards phase more.

Here, write expr for spherical wave:

$$\frac{e^{-ikr}}{r} \quad (- \text{ sign is converging})$$

shift center from $r=0$ to $z=f$

$$r \to \pm\sqrt{x^2+y^2+(z-f)^2}$$

Now make paraxial approximation: $|z-f| \gg x, y$

$(x, y \le D/2$ where D = lens dia$)$

$$r = \pm(z-f)\sqrt{1+\frac{x^2+y^2}{(z-f)^2}} \approx \pm(z-f)\left(1+\frac{1}{2}\frac{x^2+y^2}{(z-f)^2}\right)$$

Paraxial spherical wave is

$$\exp\left(\mp ik(z-f) \mp ik\frac{1}{2}\left(\frac{x^2+y^2}{z-f}\right)\right)$$ choose sign to give converging wave at $z=0$:

At $z=0$,

$$\therefore\ E_{in}(x,y) = E_0\, e^{-ik(x^2+y^2)/2f}\, e^{-ikf}$$

Now calc Fresnel integral: let $T(x,y)$ be transm. profile (e.g. of slit)

$$E_{diff} = \frac{e^{ik(R-f)}}{i\lambda R}\iint T(x,y)\, E_0\, e^{-ik(x^2+y^2)/2f}\, e^{+ik(x^2+y^2)/2R}\, e^{-ik(xX+yY)/R}\, dx\,dy$$

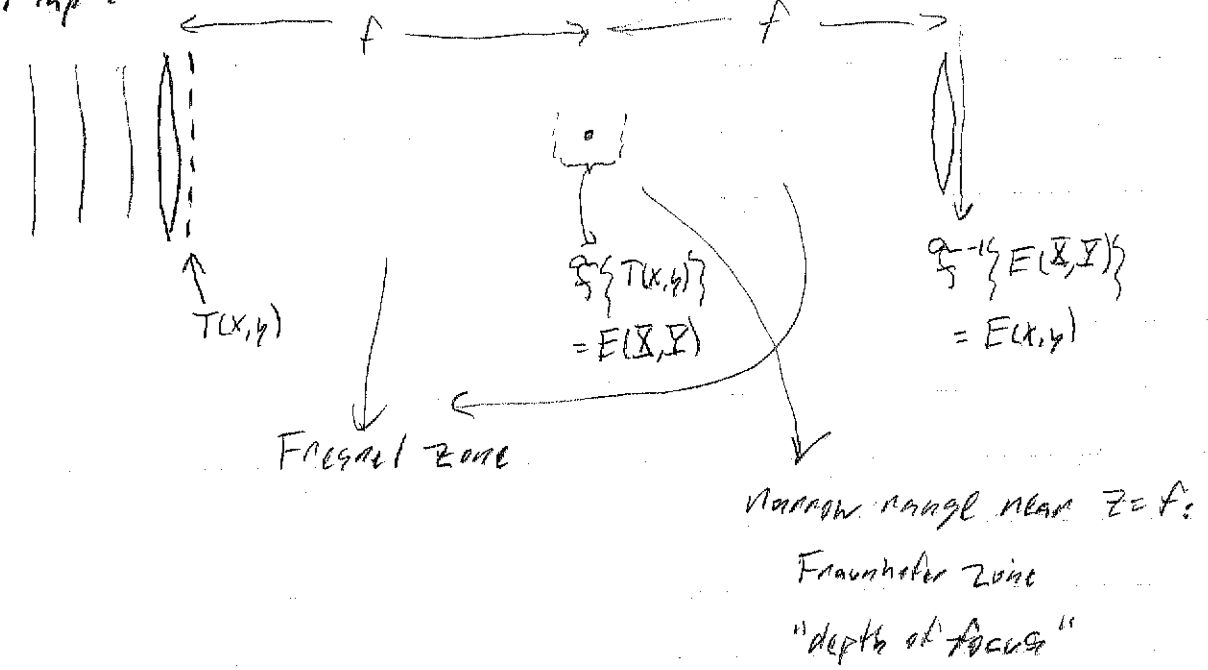
Evaluate at $R = f$: quadratic phase terms cancel!

$$E_{diff}(X,Y) = \frac{1}{i\lambda f}\iint T(x,y)\, E_0\, e^{-ik\left(\frac{xX}{R}+\frac{yY}{R}\right)}\, dx\, dy$$

$$= \frac{E_0}{i\lambda f}\, \mathcal{F}_{xy}\{T(x,y)\}$$

now spatial frequency variables are $\beta_x = \frac{kX}{f}$, $\beta_y = \frac{kY}{f}$

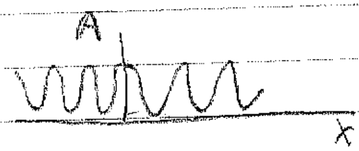
Map:



Spatial frequencies.

cosine transmission function:

$$A(x) = \tfrac{1}{2}(1 + \cos(k_{x0}x))$$



at lens focus:

$$E_{dif}(X) \propto \int \tfrac{1}{2}(1 + \cos(k_{x0}x))\, e^{-i\beta_x x}\, dx$$

$$= \mathcal{F}\left\{ \tfrac{1}{2}(1 + \cos(k_{x0}x)) \right\}$$

$$= \tfrac{1}{2}\, \mathcal{F}\left\{ e^{0} + \tfrac{1}{2}\left(e^{ik_{x0}x} + e^{-ik_{x0}x}\right) \right\}$$

$$= \tfrac{1}{2}\left\{ 2\pi\, \delta(\beta_x) + \pi\left(\delta(\beta_x - k_{x0}) + \delta(\beta_x + k_{x0}\right) \right\}$$

$$\beta_x = +kX/f$$

$$E_{dif}(X) \propto \delta\left(\tfrac{k}{f}X\right) + \tfrac{1}{2}\delta\left(\tfrac{k}{f}X + k_{x0}\right) + \tfrac{1}{2}\delta\left(\tfrac{k}{f}X - k_{x0}\right)$$