Homework 6
PH462 EM Waves and Optical Physics

due 13 October 2006 by 5pm
posted: 7 October 2006
Clarifications: 12 October 2006

Reading: Heald and Marion (HM) chapter 6 and posted notes.

1) HM 7-9
2) HM 7-10. See eqn 1.95 for how to calculate the surface currents.
3) HM 7-13
4) HM 7-14
5) Suppose a slab dielectric waveguide is asymmetric: there is a different index of refraction on each side of the core (with $n_1 < n_2$ and $n_3 < n_2$). What is the smallest value of a/λ for which there is only one propagating mode, assuming that $n_2 - n_1 << n_2 - n_3$? (Let 2a be the full width of the well.) You do not have to explicitly solve for the modes to get the answers to this problem. You may find it helpful to solve this by analogy to the equivalent Schrödinger's equation problem:
   - what is the equivalent potential?
   - what kind of behavior will the wave exhibit near the cutoff?

6) Calculate the conditions on the TM bound modes for a planar dielectric waveguide. The **cladding and core indices are n_1 and n_2,** respectively, and the walls of the core are at $x = \pm a$. Derive the conditions on the allowed *symmetric* modes:

$$\frac{\alpha\, a}{n_2^2} \tan \alpha\, a = \frac{\beta\, a}{n_1^2}$$

Inside the waveguide core, use cos(αx) for the field; outside in the cladding, use exp[-β|x|]. In this calculation, solve for the B field, since in this case $\mathbf{B} = \hat{\mathbf{y}} B_y$ only. To get the condition on the derivative of B_y across the interface, use Maxwell's equation for the curl of **B**.

7) The refractive index of a plasma is given by: $n^2 = 1 - \frac{\omega_p^2}{\omega^2}$, where $\omega_p^2 \equiv \frac{4\pi n_e e^2}{m_e}$ is the plasma frequency. The plasma frequency is associated with a collective resonance which, in the context of the refractive index, controls the response time of the electrons to the wave. In this expression, n_e is the electron number density and m_e is the electron mass. Note that if $\omega >> \omega_p$, we can write an approximate form for the refractive index: $n \approx 1 - \frac{1}{2}\frac{\omega_p^2}{\omega^2}$. In this problem, you will investigate a way to produce a waveguide for light in a plasma using a step-index model. Suppose there is a central "core" region (radius *a*) with one density, and a "cladding" region ($r > a$) with another. The net effect is that there is a step index dielectric optical fiber. Derive an expression for the maximum density difference for which the waveguide will confine a *single* mode. The production of such a density difference is possible by heating the plasma up on axis.