Homework 6 — due 15 October 2006 in class
PH462 EM Waves and Optical Physics — posted: 07 October 2006

Reading: Heald and Marion (HM) chapter 6 and posted notes.

1) HM 7-9
1) HM 7-10. See eqn 1.95 for how to calculate the surface currents.
2) HM 7-13
3) HM 7-14
4) Suppose a slab dielectric waveguide is asymmetric: there is a different index of refraction on each side of the core (with $n_1<n_2$ and $n_3<n_2$). What is the smallest value of a/λ for which there is only one propagating mode, assuming that $n_2 - n_1 << n_2 - n_3$? You may find it helpful to solve this by analogy to the equivalent Schrödinger's equation problem: what is the equivalent potential? What kind of behavior will the wave exhibit near the cutoff?

5) Calculate the conditions on the TM bound modes for a planar dielectric waveguide. The **cladding and core indices are n_1 and n_2,** respectively, and the walls of the core are at $x = \pm a$. Derive the conditions on the allowed symmetric modes:
$$\frac{\alpha\, a}{n_2^2}\tan\alpha\, a = \frac{\beta\, a}{n_1^2}$$
Inside the waveguide core, use cos αx for the field; outside in the cladding, use Exp[-β|x|]. In this calculation, solve for the B field, since in this case $\mathbf{B} = \hat{\mathbf{y}}B_y$ only. To get the condition on the derivative of B_y across the interface, use Maxwell's equation for the curl of **B**.

6) The refractive index of a plasma is given by: $n^2 = 1 - \frac{\omega_p^2}{\omega^2}$, where $\omega_p^2 \equiv \frac{4\pi n_e e^2}{m_e}$ is the plasma frequency. The plasma frequency is associated with a collective resonance which, in the context of the refractive index, controls the response time of the electrons to the wave. In this expression, n_e is the electron number density and m_e is the electron mass. Note that if $\omega >> \omega_p$, we can write an approximate form for the refractive index: $n \approx 1 - \frac{1}{2}\frac{\omega_p^2}{\omega^2}$. In this problem, you will investigate a way to produce a waveguide for light in a plasma using a step-index model. Suppose there is a central "core" region (radius *a*) with one density, and a "cladding" region ($r > a$) with another. The net effect is that there is a step index dielectric optical fiber. Derive an expression for the maximum density difference for which the waveguide will confine a *single* mode. The production of such a density difference is possible by heating the plasma up on axis.