Phys 361 Homework 12

1) Consider a long, straight, solid cylindrical wire of radius a made out of some paramagnetic material with susceptibility χ_m. The wire is carrying a uniform volume current of total magnitude *I* flowing down the length of it. In other words, it's a completely normal current-carrying wire.

Calculate the magnetic field everywhere, inside and outside the wire. Then calculate the bound surface and volume currents (if any) and the magnetization in the material. Now look at the bound and free currents separately and write the magnetic fields each of them make individually, inside and outside. The sum of these contributions should, of course, be the net field that you've already calculated.

Oh, and one last thing: Without doing any math, what would you expect the net bound current flowing down the length of this wire to be? Check and see if your expectations are met.

2) (based on Pollack and Stump 9.3)

a) Estimate the maximum possible magnetization $\vec{M}$ of normal iron. Assume the atomic dipole moment of an iron atom is due to two aligned electron spins. Maximum magnetization occurs when all of these dipoles are pointing in exactly the same direction.

b) We can get an answer for part a, and that's all well and good, but is it a lot? Let's find out. Assume you have a spherical magnet of radius 2 cm with a uniform magnetization in the $\hat{k}$ direction (the magnetization from part a). What is the strength of the magnetic field inside the magnet? This is the B that you'd get if every dipole moment in a little magnetic ball was aligned. Of course, we don't know if *that's* big or not without knowing how big a "big" field is. Root around in books or on the web and decide for yourself (and tell me): What scale of B-field constitutes a "big" one, and how does this little ball compare?

3) (based on Pollack and Stump 9.19)

As you've been discovering lately in Thermal, finding mean quantities thermodynamically involves taking weighted averages where the Boltzmann factor $e^{\frac{-U}{kT}}$ is the weighting function. So for example, in a situation where the potential energy of a particle *U* depends on *x*, the mean value of *x* for that particle at a temperature *T* would be given by $\int_0^\infty \frac{e^{\frac{-U(x)}{kT}} x dU(x)}{e^{\frac{-U(x)}{kT}} dU(x)}$ if *x* can range from 0 to infinity. Let's apply that idea here. If you're not in Thermal right now, there's a good chance that you'll be able to get by with the preamble I just provided. But if you're not sure what's going on, please feel free to come get some guidance.

a) Suppose we have a gas of magnetic dipoles of number density (number of molecules per volume) n, with each dipole having moment m_0 and each dipole being free to point in any direction. The gas is sitting in a magnetic field of constant magnitude, $B_0\hat{k}$. Show that the magnetization of the material is given by the so-called Langevin formula:

$$\vec{M} = nm_0 \coth(b - {}^1/_b)\,\hat{k}$$

Where $b = {}^{m_0B_0}/_{kT}$. Some hints: It's pretty reasonable to assume that the magnetization will end up pointing in the direction of the applied magnetic field, so start by finding the mean value of the z component of the dipole moment of any particular dipole. Also note that this is a gas. What does that say about the likely size of χ_m, and the size of μ relative to μ_0?

b) Plot the magnetization as a function of b, in units of nm_0. Briefly interpret this graph. Show that for $kt \gg m_0B_0$ the susceptibility approaches $nm_0^2\mu_0/3kT$.

c) The answer from part (a) assumes the dipoles can point in any direction, in a very classical way. Re-derive $\vec{M}$ assuming that each dipole can only either be spin up or spin down (that is, it can either be pointing along the applied B-field, or against it).

Note that if you have a discrete set of states, you take averages with sums instead of integrals. And if there are only two possible states, it's a pretty short sum. There should be a tanh in your answer.