PHGN 462 Homework 6

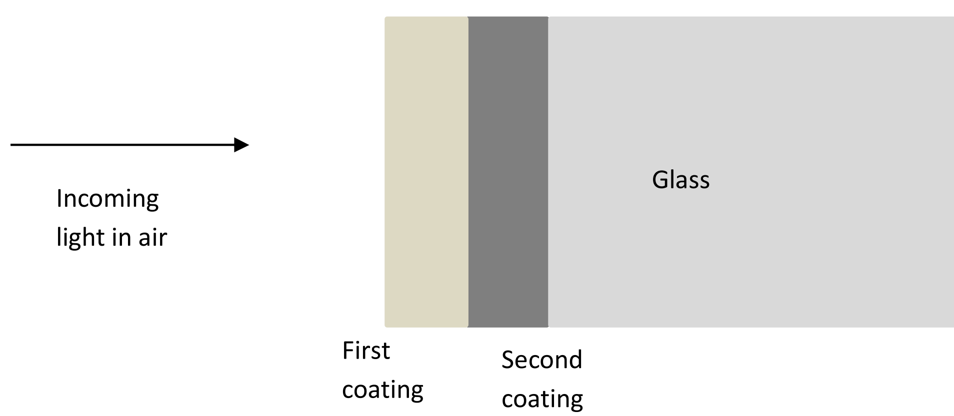
1) (Based on Pollack and Stump 13.5 – also, read problem 2 before you start this one)

Consider light incident normally on a plate of glass with thickness *a*, with air on either side of the plate. You can treat the air as having index of refraction 1, and the glass as having index 1.5.

a) Write **E**(x,t) and **B**(x,t) in the three regions, with x being the propagation axis and $\hat{j}$ being the direction in which the E-field is polarized. Write the four boundary conditions on the wave amplitudes.

b) Solve for the transmission coefficient *T*, i.e., the ratio of transmitted intensity to incident intensity. Plot *T* as a function of *k**a*, where *k* is the magnitude of the incident wave vector. While it is technically possible to do this analytically for this problem, I strongly recommend you do it numerically. Interpret the resulting graph physically, and tell me why it has the period it has.

2) If you set up problem 1 in a really general, symbolic way using good Mathematica code, you have the foundation you need to solve more complicated multilayer problems fairly quickly. Let's try something that's pushing in the direction of a realistic industry problem.



The above diagram is a crude picture of glass with two dielectric coatings on it. It could be a lens, a mirror, or a bunch of other things. Let's use materials that are common in actual practice. The substrate is BK7 glass, very common in optics on account of being relatively low-dispersion, which means that different colors of visible light see more-or-less the same index of refraction, 1.52. One of the coatings is made of MgF_2 (magnesium fluoride), with approximate index 1.37 for visible wavelengths. The other is made of Al_2O_3 (aluminum oxide), with index 1.77. Both of these materials are very durable and scratch-resistant, and make great coatings. As a side note, aluminum oxide is the basis for gems such as sapphire and ruby.

This is a physical setup with four regions and three interfaces. Note that we're considering the output end to be in the glass, *not* the air on the far end of the glass. We're going to solve for the transmission coefficient of this system as a function of incoming wavelength and coating thickness, considering the possibility that the coatings might be of unequal thickness.

a) Write down **E**(x,t) and **B**(x,t) in the four regions, letting $\hat{j}$ be the polarization direction. Then write down the six relevant boundary conditions on those fields. This should be at most a minor extension of the work you did in (1a).

b) Use Mathematica (or whatever) to get the transmission coefficient for the system as a function of thicknesses a and b and incident wavelength λ. Repackage the variables in whatever way you find convenient (for example, if you'd rather work with wavevector k, or some defined combo like $x = k*a$, go nuts). Pick some interesting, specific values for a, b, and λ and graph T as a function of λ (or k, or $k*a$, or whatever you find appropriate). This, also, should be pretty much re-use of what you did in problem 1.

c) Now let's get interesting. People tailor multilayer dielectric coatings such as this one towards specific design constraints. For example, a client might need an optic that simultaneously minimizes reflection of 532 and 650nm light, presumably cutting out other wavelengths in the process. A company that makes custom coated optics wouldn't limit themselves to two layers using two materials, but for the sake of keeping it "easy" let's say we have the previously specified system and we want to choose values for a and b that minimize reflections at 532nm and 650nm. Don't sweat what happens at other wavelengths.

Optimization is kind of a tricky business, and I'm not going require that you do it a certain way. This is where you get to exercise a little creativity. Find the best possible a and b for the given parameters any way you please.

Make sure that when you're writing up this problem, you tell me what you did and show me a graph of your optimal solution. You get full credit for coming up with any remotely decent solution and a clear explanation of what you've done.

(bonus opportunity) If you want a shot at bonus credit, do something extra snazzy with your solution and/or analysis, where I'm intentionally underspecifying "extra snazzy." The snazzier the better. Have fun with it, investigate something you actually care about, learn fancy Mathematica tricks, whatever. If you're shooting for the bonus credit, turn in your homework as usual, but also email your code and a scan of your handwritten stuff directly to me. I'll evaluate the bonus myself. As a reminder, for something to qualify for bonus credit, it should represent effort *significantly* in excess of a solution that would receive full regular credit. Remember, I apply bonus credit to your grade *after* I make the letter grade cuts – that way it's a true bonus, and you shouldn't feel any pressure to go after it just to keep up with other people.