Physics 311, Fall 2005

Introduction to Mathematical Physics

As of Spring 2005, GPGN 249 and GPGN 306 have been folded into PHGN 311; as you know already, the two sections of this course meet at 9 AM [section B, Snieder]] and 10 AM [section A, Scales] MWF.

The two instructors will have staggered office hours for maximum availability. These will probably consist of two-hour 'problem session' blocks in which students will work homework problems at the blackboard.

The physics Field Session has in the past been a prerequisite for PHGN 311 because it contains a fairly serious exposure to *Mathematica* , which we will use extensively for visualization, and for routine calculations which would otherwise be tedious. This fall we will spend a couple of days on *Mathematica* for those who have not had this exposure. A site license at the School of Mines guarantees this wonderful tool will be commonly available on campus, not to mention on the physics and geophysics networks. A number of *Mathematica* notebooks will be distributed via the (physics department) Web site for this course, at the URL `http://ticc.mines.edu/wiki/index.php/Math_Physics`. The *modus operandi* will be that *Mathematica* should **NOT** be used in a given problem or problem set unless specifically indicated, in order that you avoid a hideous and unseemly dependence on a computer algebra program and spend 28 days in an expensive clinic. **BUT:** By all means use *Mathematica* to *check* your answers (but don't turn in your checks).

Course Text: *Mathematical methods in the physical sciences*, **Third Edition**, Mary L. Boas (2006, John Wiley & Sons). ISBN 0-471-19826-9.

Course Objectives: This course will provide the mathematical tools necessary for later courses in physics and geophysics at the junior and senior undergraduate level.

Students completing this course satisfactorily will be able to

1. Identify dimensionless variables to extend the scope of a calculation
2. Mingle text explanations, simple equations, and graphics in *Mathematica* Notebooks (useful in many courses and in Real Life)
3. Manipulate simple functions of complex numbers
4. Solve sets of linear equations and find eigenvalues and eigenvectors using the tools of linear algebra, matrices, and determinants
5. Be comfortable with the treatments of functions as vectors in an abstract vector space with a peculiar 'dot product'
6. Be able to identify when to use discrete or continuous Fourier transforms
7. Be able to use Fourier (and Laplace!) transforms to solve simple problems and understand qualitatively how a function and its Fourier transform are related
8. Recognize some of the 'special functions' which emerge when we separate variables in Cartesian, spherical, and cylindrical coordinate systems.
9. Be able to use the Dirac and Kronecker delta functions in routine calculations
10. Be comfortable with 'series solutions' to second order ordinary differential equations.

Course Outline

- Using *Mathematica* : getting started, preparing a document, getting help, resources, printing ecologically when submitting homework problems, and warnings.
- Dimensionless quantities in physics: why they arise, how to take advantage of them. Using the 'Buckingham pi' theorem.
- A quick review of common series.
- Review of complex numbers and simple functions of complex variables.
- Linear algebra: the vector spaces we use to describe n-dimensional real *or complex* spaces. Scalar products, orthonormal sets of vectors. Applications in physics. Matrices: special forms of matrices and operations on them; determinants. Using matrices to solve sets of linear equations; eigenvectors and eigenvalues and scads of applications. Physical origins of tensors and relation to matrices.
- Fourier sums and the transition to Fourier integrals. Applications to quantum mechanics, diffraction. Solutions of simple partial differential equations (with boundary conditions) using Fourier transforms. Three-dimensional Fourier transforms for spherically-symmetric problems and examples. Correlation functions, convolutions and applications in optics and image processing.
- A review of first and second-order differential equations. Series solutions to second-order differential equations: Wronskians, the recursion relation, and determining whether two solutions are linearly independent.
- A smattering of 'special functions': the Dirac δ function and some common 'representations' of it.
- Quick review of vector calculus in common coordinate systems: applications of Gauss' and Stokes' Theorems and the scalar and vector potentials. Laplace's and Helmholtz's equations and a quick introduction to separating variables in two and three dimensions.

Homework: Problem sets will be assigned *weekly*, with sets due by 5 PM on the due date. You are encouraged (but not required) to work independently on these—unfortunately, when people collaborate there is a clear distinction in exams between those who did the work and those who mostly observed.

Examinations: There will be one mid-term exam and one final exam. These will be two hours each and open-book.

Grade: Homework: 33%; mid-term test 33%, Final: 34%.

Supplemental References: Texts roughly comparable to course text

1. *A Guided Tour of Mathematical Methods for the Physical Sciences*, Second Edition, Roel Snieder (Cambridge University Press, 2004). Looks very entertaining, with examples drawn from geophysics.
2. *Advanced Engineering Mathematics*, Second Edition, Dennis G. Zill and Michael R. Cullen (Jones and Bartlett, 2000). Skimpy on proofs?

3. *Advanced Engineering Mathematics*, Sixth Edition, C. Ray Wylie and Louis C. Barrett (McGraw-Hill 1995). Used it but didn't like it much.

4. *Essential Mathematical Methods for Physicists*, Hans J. Weber and George B. Arfken (Elsevier/Academic Press, 2004). This is a stripped-down version of the book by Arfken & Weber below.

Vector calculus:

1. *Vector Calculus*, Fourth Edition, Jerrold E. Marsden and Anthony J. Tromba (W. H. Freeman, 1996). Recommended by KvW.

Higher level:

1. *Mathematics for Physicists*, Susan M. Lea (Thomson Brooks/Cole 2004). This text is being used in the senior/first-year graduate mathematical physics course.

2. *Mathematical Physics*, Eugene Butkov (Addison-Wesley, Reading, Mass, 1968). This is a good text, used for many years for PHGN412/511, but a little dated. It suffers from the belief of the author that nothing should be remembered out of context, so there are *no equation numbers.*

3. *Mathematical Methods for Physicists*, 5th Edition, George B. Arfken and Hans J. Weber (Academic Press, San Diego, 2001); ISBN 0-12-059825-6 (hardback). I used the 4th edition two years ago, but feel now that this book is better as a reference than as a text.

4. *Mathematical Methods for Physicists and Engineers*, Royal Eugene Collins, 2nd Corrected Edition (Dover Publications, Meneola, NY, 1999); ISBN 0-486-40229-0.

5. *Mathematics of Classical and Quantum Physics*, Frederick W. Byron, Jr., and Robert W. Fuller, Dover Publications, New York, 1992; two volumes bound as one); ISBN 0-486-67164-X. Such a deal—a previous classic.

6. *Mathematical Methods of Physics*, Jon Mathews and R. L. Walker (W. A. Benjamin, Menlo Park, CA, 1970); ISBN 0-805-37002-1. This is a wonderful but demanding book whose authors also have good taste. There is a smattering of numerical analysis in one chapter.

7. *Methods of Theoretical Physics*, Philip M. Morse and Herman Feshbach (McGraw-Hill, New York, 1953). Pray you never have to delve into this book, which contains much more than you probably want to know.

Supplemental References: symbolic mathematics using *Mathematica*

1. *A Physicist's Guide to Mathematica*, Patrick T. Tam (Academic Press, San Diego, 1997). Looks okay.

2. *Mastering Mathematica: Programming Methods and Applications*, John Gray (AP Professional, Boston, 1994).

3. *Differential Equations with Mathematica*, Second Edition, Martha L. Abell and James P. Braselton (Academic Press, San Diego, 1997).

Supplemental References: Numerical Mathematics

1. *Numerical Recipes* [in C or Fortran], Second Edition, William H. Press, Saul A. Teukolsky, William T. Vetterling, and Brian P. Flannery (Cambridge University Press, 1992). Remains a wonderful reference to get your feet wet in most aspects of numerical analysis.

2. *Handbook of Mathematical Functions: With Formulas, Graphs, and Mathematical Tables*, Edited by Milton Abramowitz and Irena A. Stegun, (Dover Publications, New York, 1965?). Still a classic.