Lecture 21 Shadowitz 11.1

Review:

curl	divergence
$\vec{\nabla} \times \vec{E} = 0$	$\vec{\nabla} \cdot \vec{E} = \rho/\epsilon_0$
$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$	$\vec{\nabla} \cdot \vec{B} = 0$
$\vec{\nabla} \times G = -\frac{\partial \vec{B}}{\partial t}$	$\vec{\nabla} \cdot \vec{G} = 0$

G is the Faraday field.

$$\text{Let } \vec{E}_{tot} = \vec{E} + \vec{G}$$

$$\vec{\nabla} \cdot \vec{E}_{tot} = \vec{\nabla} \cdot (\vec{E} + \vec{G}) = \vec{\nabla} \cdot \vec{E} + \underbrace{\vec{\nabla} \cdot \vec{G}}^{0} = \rho/\epsilon_0$$

$$\vec{\nabla} \times \vec{E}_{tot} = \vec{\nabla} \times (\vec{E} + \vec{G}) = \underbrace{\vec{\nabla} \times \vec{E}}_{0} + \vec{\nabla} \times \vec{G} = -\frac{\partial \vec{B}}{\partial t}$$

Since the G field generates a force on a charge that is the same as experienced by an E we rewrite G=E

$$\vec{F} = q\underbrace{(\vec{E} + \vec{G})}_{=} + q\vec{v} \times \vec{B}$$

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

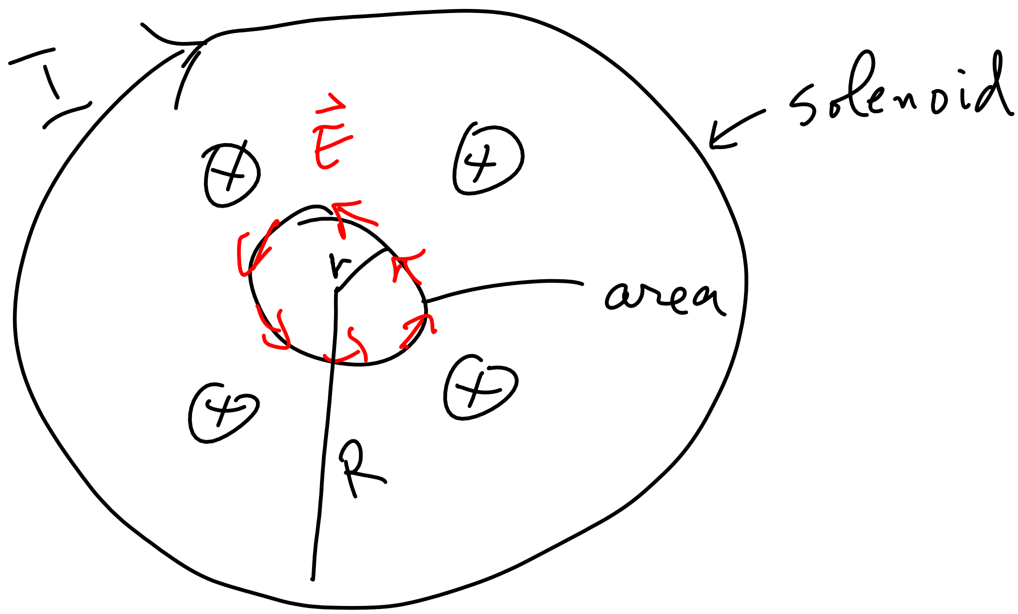
InkSurvey

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\int \vec{\nabla} \times \vec{E} \cdot d\vec{a} = -\frac{\partial}{\partial t} \int \vec{B} \cdot d\vec{a} = -\frac{\partial}{\partial t} \Phi_m$$

$$\parallel$$

$$\oint \vec{E} \cdot d\vec{r} = -\frac{\partial}{\partial t} \int \vec{B} \cdot d\vec{a}$$

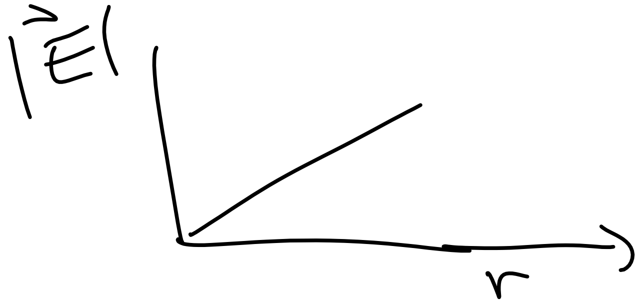


$$E\, 2\pi r = -\frac{\partial}{\partial t} B \pi r^2 = -\pi r^2 \frac{\partial B}{\partial t}$$

$$\vec{E} = -\frac{r}{2} \frac{\partial B}{\partial t} \hat{\varphi} \qquad B = \alpha t$$

$$\parallel$$

$$\mu_0 n I(t)$$



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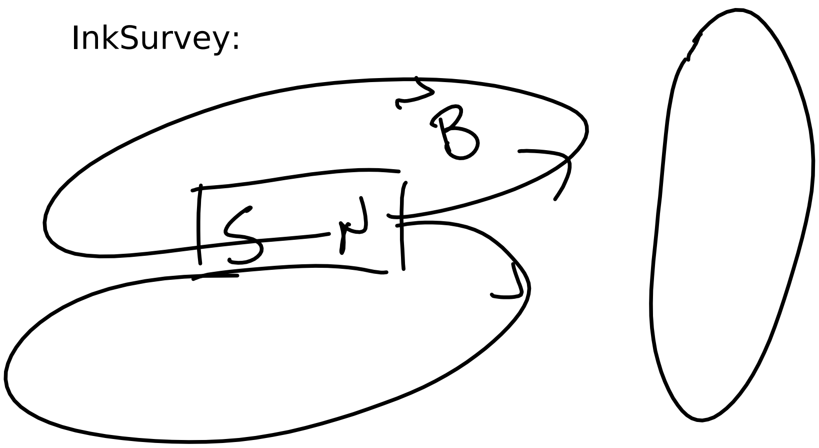


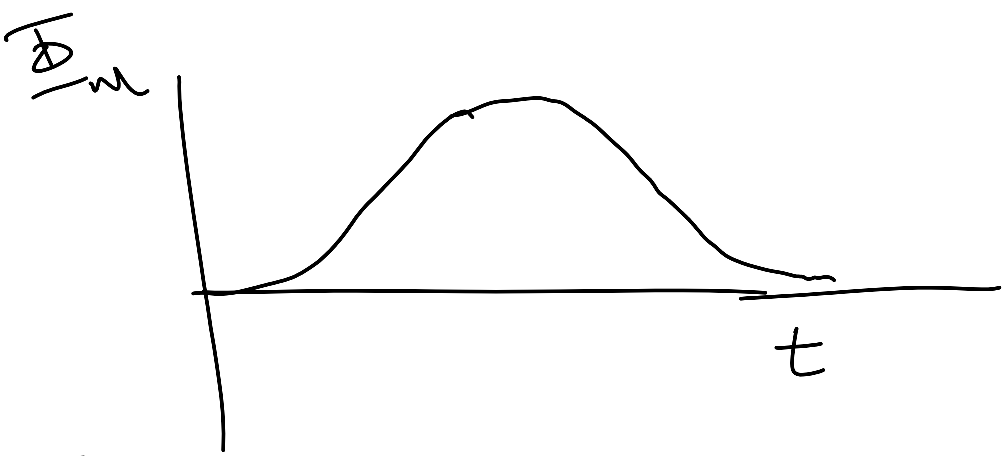
Knee-jerk thinking

no definitions or
fundamental eqns used

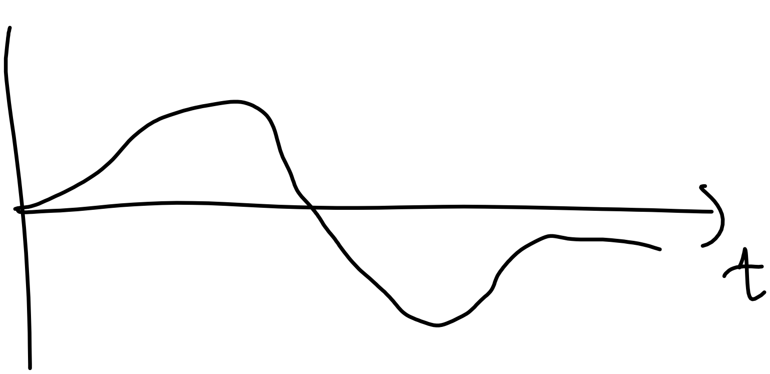
Analytic thinking

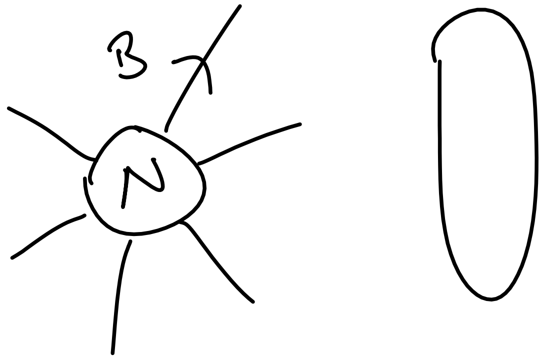
$\oint \vec{E}\cdot d\vec{r}$ generates current

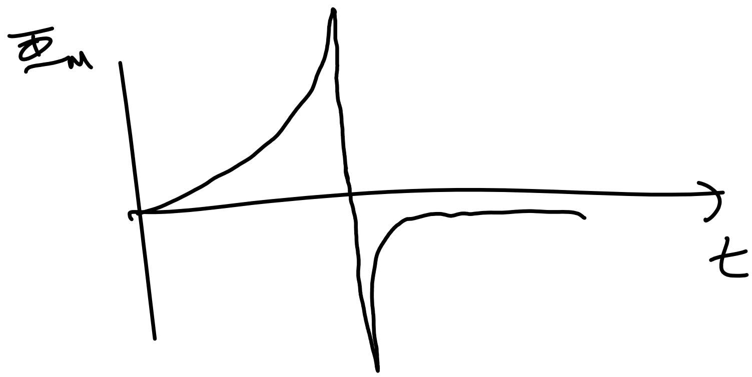
$$\left|\oint \vec{E}\cdot d\vec{r}\right| = \frac{d\Phi_m}{dt}$$



$I \propto \frac{d\Phi_m}{dt}$







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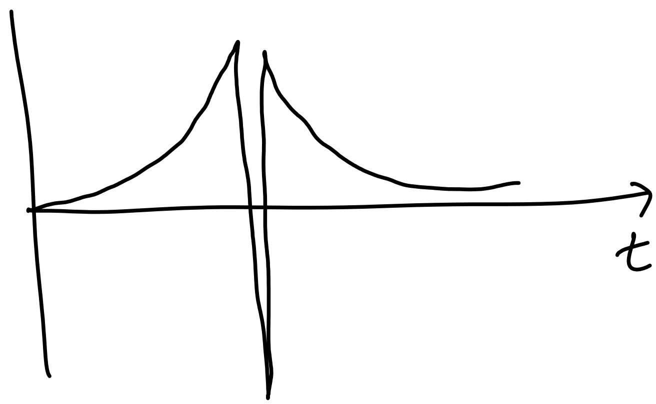


Muddiest points

- How do you use Faraday's law at a point?

Integral form

$$\oint \vec{E} \cdot d\vec{r} = -\frac{\partial}{\partial t} \int \vec{B} \cdot d\vec{a}$$

Differential form

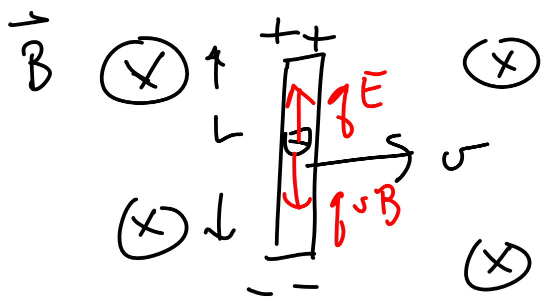
$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Given E near a fixed point its curl is the time rate of chage of B at that point.

What does the partial derivative mean?

It means to fix all other variables. In Faraday's law x,y,z are fixed while time varies. That is, we are not moving in x,y,z while using Faraday's. We are fixed in position.

How do you calculate the voltage if the rod accelerates through a B?



$$qvB = qE$$

$$E = vB$$

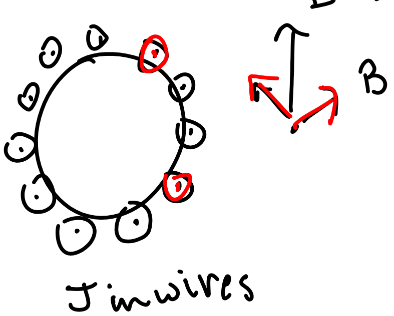
$$\Delta V = -\int \vec{E}\cdot d\vec{r} = EL$$

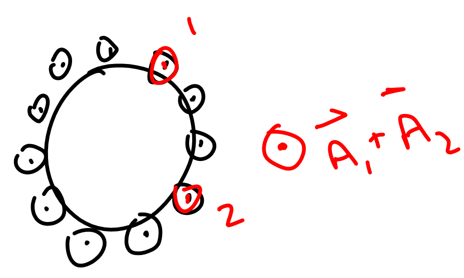
We have not determined how to work with accelerating charges. The "quasistatic approximation means that charges are moving slowly enough to neglect the effects of acceleration. Then just replace v by a t.

How do you determine the direction of A given J?

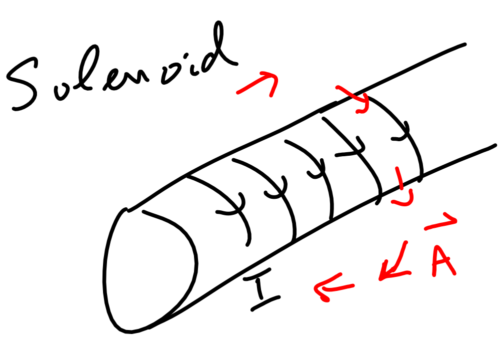
For symmetric current distributions A and J are in the same direction. This is like using Biot-Savart to find B knowing currents in a symmetrical situation.

B is tangential





A from wire with surface current



Questions about hmwk will be addressed in the posted solns.

Is it possible to create a constant E with a B?

For the solenoid problem above just make sure $\frac{\partial B}{\partial t} = \text{constant}$

$$B = \alpha t \to \infty \text{ as } t \to \infty$$

Other ways to measure A? Should I find A from B?

The math to determine radiation will start with A and V NOT E and B. We measure radiation consistent with this and not consistent when starting with E and B.

Still confused about the skydiver analogy.

$$J = \rho v \begin{cases} \text{skydivers} \\ \text{charges} \end{cases}$$

$$\vec{J} = \begin{cases} \sigma \vec{g} & \text{skydivers } \sigma \propto \text{air pressure} \\ \sigma \vec{E} & \text{charges } \sigma \text{ material} \end{cases}$$

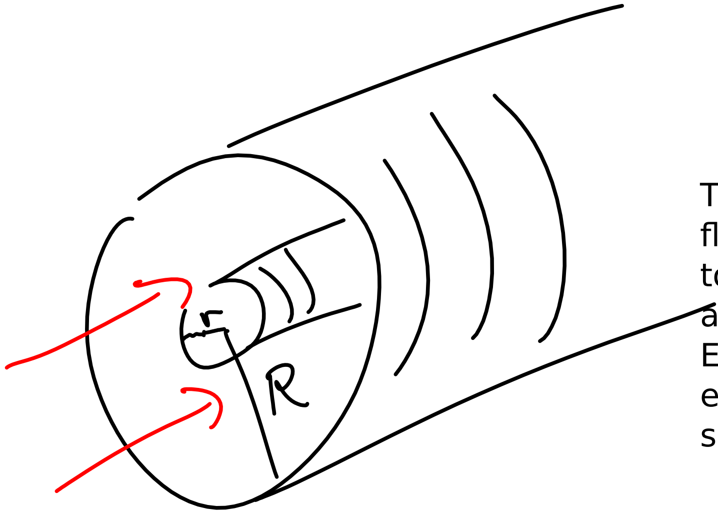
small σ large σ for constant ρ

Changing B generates a current which creates another B. Where does this process end?

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

What B is this? This is the TOTAL B from all currents.

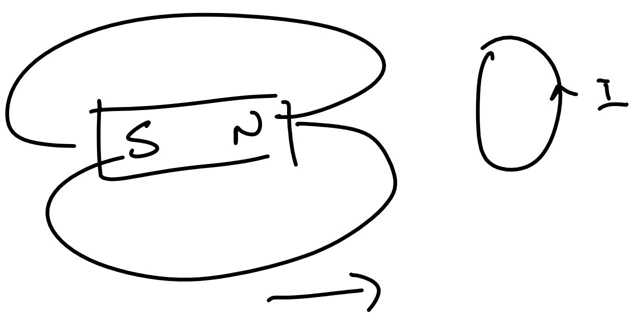
$B_R = \mu_0 n_R I(t)$

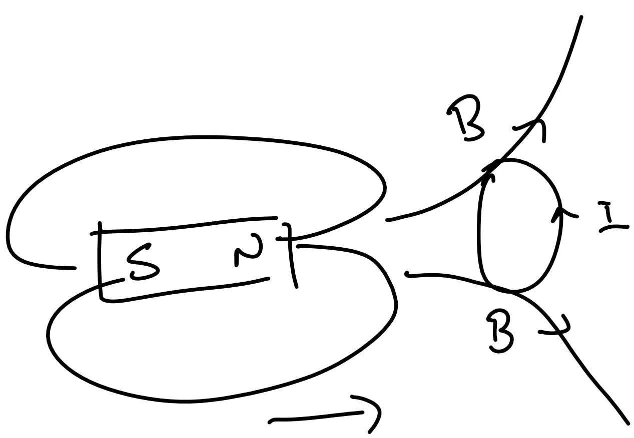


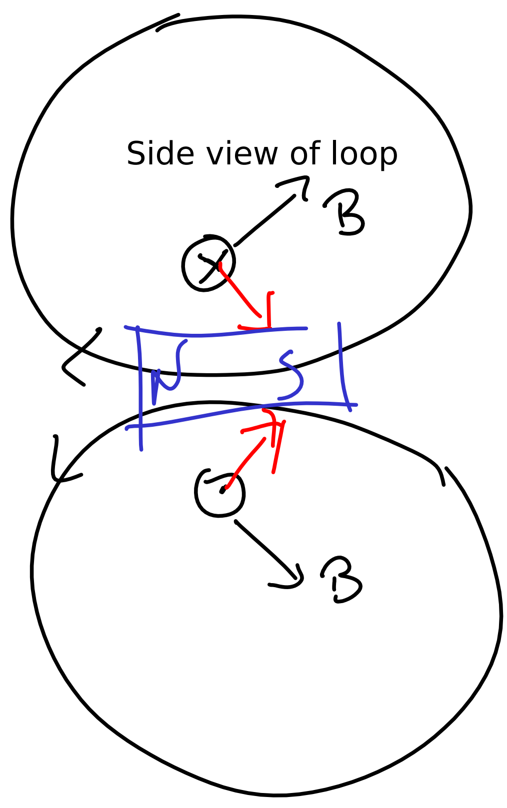
The outer solenoid generates a changing flux in the inner solenoid causing a current to flow. The inner solenoid then generates a an extra changing B which causes an extra E both in the inner and outer solenoids. This extra E then affects the currents in the solenoids etc.

The minus sign in Faraday's law is called Lenz's law.

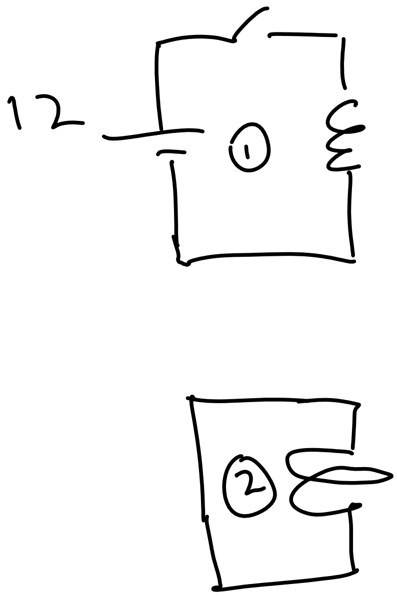
The electric field is generated in a direction which will attempt to cause a current to flow which opposes the changing flux.

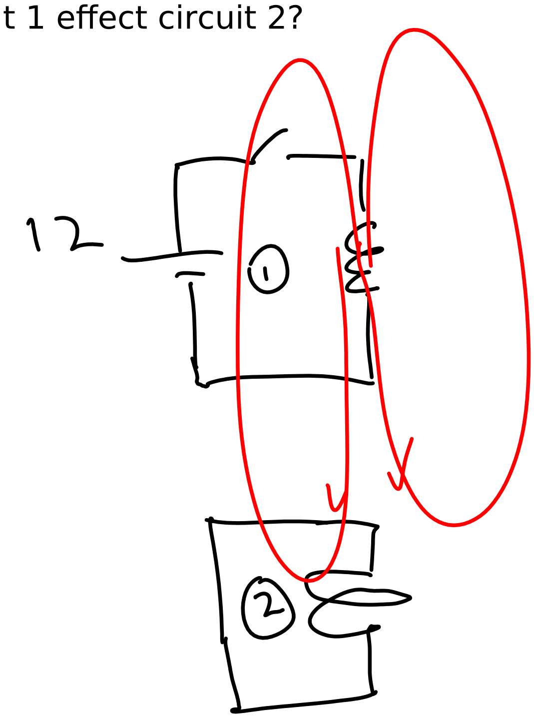






How does circuit 1 effect circuit 2?





(causal/creative) What fundamental law governs this behavior?

$$\Phi_m^{2\text{ due }1} \propto \;?$$

$$\Phi_m^{2\text{ due }1} \propto I_1$$

$$\boxed{\Phi_m^{2\text{ due }1} \equiv M_{21} I_1}$$

defn of M

$$\oint \vec{E}_2 \cdot d\vec{r} = M_{21} \frac{dI_1}{dt}$$

assuming M_{21} constant

mutual inductance

$\oint \vec{E}_2 \cdot d\vec{r}$ around one loop $= \Delta V$ or Emf

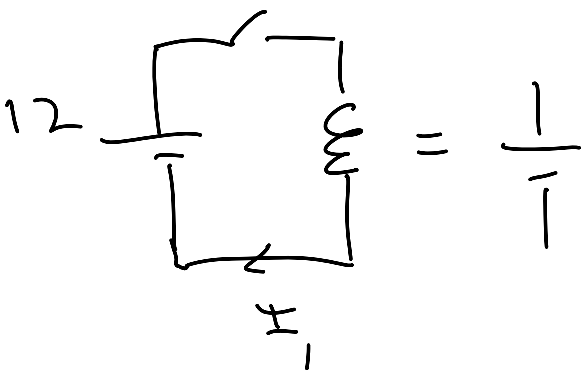
For N loops in circuit 2, $\Delta V_{tot} = \Delta V_1 + \Delta V_2 + \cdots$

$= N \Delta V$

(analogous) What analogous relation is there in electrostatics?

$M = \frac{\Phi_m}{I}$ $\quad \frac{\text{flux}}{\text{amp}}$ $\qquad C = \frac{Q}{V}$ $\quad \frac{\text{coul}}{\text{volt}}$

(congruous) How do I calculate M?



$\Phi_1 \propto I_1$

$\Phi_1 = L I_1$ $\qquad$ Emf $= -\frac{d\Phi_1}{dt} = -L\frac{dI_1}{dt}$

Questions about this model?

-(congruous)why does this have a non-zero curl?

-(congruous) will there be a time dependence in E?

--(congruous) Is E a neg time der of A.

-(congruous) what does the minus sign signify?

-(cong) continuity eqn.

-(congr) What does Stokes theorem tell us?

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Since this a non-conservative field maybe we should call it G rather than E. Remember the curl of E is zero.

$$\vec{\nabla} \times \vec{G} = -\frac{\partial \vec{B}}{\partial t}$$

$$\int \vec{\nabla} \times \vec{E} \cdot d\vec{a} = -\frac{\partial}{\partial t} \int \vec{B} \cdot d\vec{a} = -\frac{\partial}{\partial t} \Phi_m$$

$$\oint \vec{E} \cdot d\vec{r} = -\frac{\partial}{\partial t} \int \vec{B} \cdot d\vec{a}$$