PHGN 462 Homework 5

1) (From Pollack and Stump 11.13)

Consider the electromagnetic field

$$\vec{E}(x,y,t) = E_0 \cos\left(\frac{\pi x}{L}\right)\cos\left(\frac{\pi y}{L}\right)\sin(\omega t)\,\hat{k}$$

$$\vec{B}(x,y,t) = B_0\left[-\cos\left(\frac{\pi x}{L}\right)\sin\left(\frac{\pi y}{L}\right)\hat{\imath} + \sin\left(\frac{\pi x}{L}\right)\cos\left(\frac{\pi y}{L}\right)\hat{\jmath}\right]\cos(\omega t)$$

This is an example of a standing wave. Standing waves in cavities show up in a lot of practical places; not the least of which is a laser cavity. This one in particular might be a wave in a long rectangular box stretching out in the z direction, with walls of length L in the x and y directions.

a) Show that this field satisfies the Maxwell equations in vacuum if $\omega = \frac{\sqrt{2}\pi c}{L}$ and $B_0 = \frac{E_0}{\sqrt{2}c}$. Notice in particular that our old familiar $E = cB$ doesn't apply here – that result was derived for plane waves, and this isn't one.

b) Sketch the E and B fields. These are moderately complicated fields and are not trivial to draw. I'd like you to give particular thought to how best to visually represent those fields in this situation. There is no one accepted standard. If you can get a software package to give you good results, then feel free to submit that.

(problem 2 on next page)

2) (From Pollack and Stump 13.23)

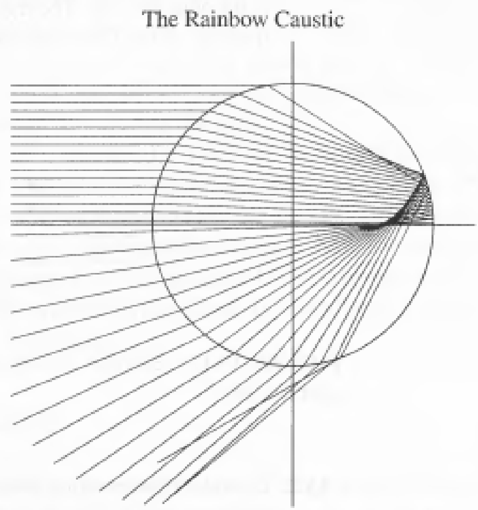

FIGURE 13.13 Illustration of how light refraction and reflection in a water droplet causes a rainbow. Light incident from the left is refracted on entering the spherical drop, reflects partially from the back surface, and is refracted on leaving the front surface. (Only rays incident above the midplane are shown.) The caustic—the region where the exiting rays are most concentrated—is the rainbow. The colors of the rainbow are the result of dispersion; the angle of the caustic varies with wavelength.

Figure 13.13 shows the light scattering process that creates the primary rainbow. Light rays at varying impact parameter refract into a spherical water drop, reflect from the back surface, and refract out of the drop. (In the figure, only the rays entering the upper half of the drop are shown. The rays shown are the rays that would reach the ground.) At a scattering angle of 42 degrees there is a concentration of scattered rays, called the caustic, and that somewhat more intense scattered light is the rainbow.

a) Explain why the ordering of colors (ROYGBIV) is red at the outer edge of the arc, and violet at the inner edge.

b) A secondary rainbow, in which the order of colors is reversed, is sometimes visible at a higher angle than the primary. Explain this second arc.

c) Explain why the area inside the primary rainbow is brighter than the area outside.

Note that “explain” doesn’t mean copy text from Wikipedia with a couple words changed. I want to see diagrams and geometry and stuff. Roughed out, at least.