Note Title 9/25/2006

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Exam oct. 11

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}$$

$x + y = \alpha$

$y = \beta$

$2y = \gamma$

$\boxed{\beta = \gamma = 0 \quad \alpha = 1}$ RHS is in the col. space of A

$x = 1$

$y = 0$

$y = 0$

$\boxed{\alpha = 0 \quad \beta = 0 \quad \gamma = 1}$ not in the col. space of A

$x + y = 0$

$y = 0$

$y = 1/2$

$Ax = y \qquad y \notin$ col. space (A)

$Ax - y \neq 0$

$Ax - y = r$

$A^T(Ax - y) = A^T r = 0$

$$Ax = y$$

$$A^T A x = A^T y \quad \text{normal equations}$$

$$\downarrow$$

$$x_{LS}$$

$\|\vec{x}\|$ Norm of x, "length"

$$\|\vec{x}\|_{\ell_p} = \left(\sum_{i=1}^{N} |x_i|^p \right)^{1/p}$$

$$\|\vec{x}\| = \|\vec{x}\|_{\ell_2} = \left(\sum_{i=1}^{N} |x_i|^2 \right)^{1/2}$$

$$\|\vec{x}\|_{\ell_2} = \sqrt{\vec{x} \cdot \vec{x}} = \sqrt{(\vec{x}, \vec{x})}$$

$$\|\vec{x}\|^2 = (x, x)$$

$$\begin{bmatrix} | \\ | \\ | \\ | \\ | \end{bmatrix} x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix}$$

$A \in \mathbb{R}^{N \times 1}$ $\quad x \in \mathbb{R}^1$

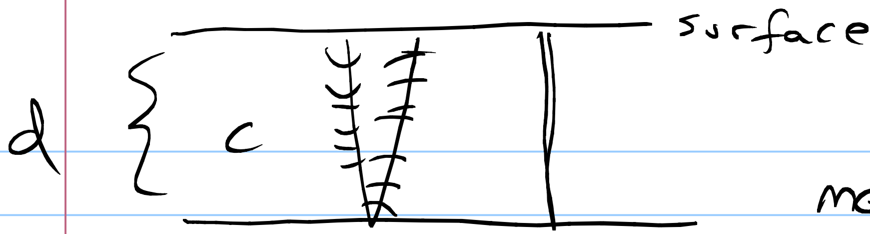
$A^T = [1, 1, \dots \; 1]$

$A^T A = N \qquad A^T y = \sum_{i=1}^{N} x_i$

$$A^T A x = A^T y$$

$$N \cdot x = \sum_{i=1}^{N} x_i$$

$$x_{LS} = \frac{1}{N} \sum_{i=1}^{N} x_i$$



measure t

$$t = f(c,d) = 2\frac{d}{c} \qquad \frac{2}{c} = \frac{1}{C} \qquad C = \frac{c}{2}$$

data, rhs, $t \in \mathbb{R}^1$

unknown vector $\begin{bmatrix} d \\ c \end{bmatrix} \in \mathbb{R}^2$

$t = \frac{d}{c}$ linearize about d_0, c_0

$$t = t_0 + \left.\frac{\partial f}{\partial d}\right|_{d_0, c_0} (d - d_0) + \left.\frac{\partial f}{\partial c}\right|_{d_0, c_0} (c - c_0)$$

$$t = \frac{d}{c} \qquad \frac{\partial f}{\partial d} = \frac{1}{c} \qquad \frac{\partial f}{\partial c} = -\frac{d}{c^2}$$

$$\frac{t - t_0}{t_0} = \frac{1}{c_0}(d - d_0) - \frac{d_0}{c_0^2}(c - c_0)$$

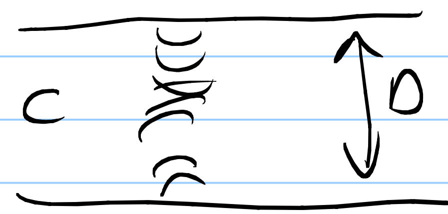
$$\frac{\delta t}{t_0} = \frac{\delta d}{d_0} - \frac{\delta c}{c_0}$$

$$T = O - C$$

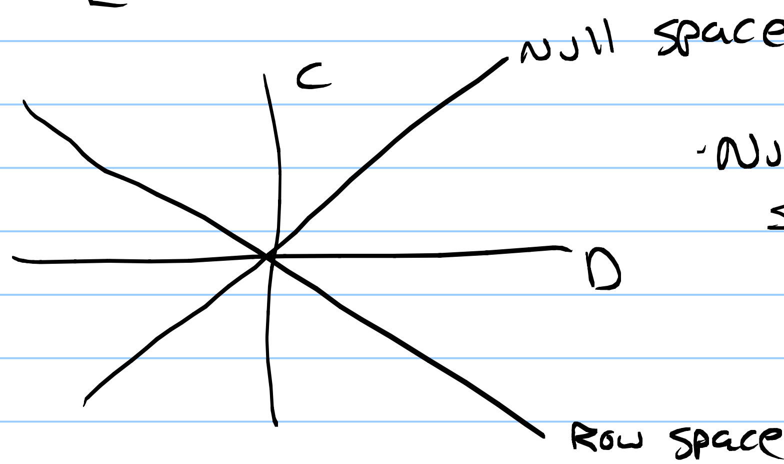
Datum $T \in R^1$

$\begin{bmatrix} D \\ C \end{bmatrix} \in R^2$ $\quad A = [1, -1]$

$[1, -1] \cdot \begin{bmatrix} D \\ C \end{bmatrix} = T$



$[1, -1] \begin{bmatrix} x \\ x \end{bmatrix} = 0$



· Null space $\perp$ row sp
subspaces of R^2

$\text{Dim}(N(A)) = 1$

$\text{Dim}(\text{RowSpace}(A)) = 1$

$\text{Dim}(\text{column space}(A)) = 1$

$\text{Dim}(N(A^T)) = 0$

$\begin{bmatrix} 1 \\ -1 \end{bmatrix} x = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$R(A)$ = # of linearly indep. colums

= # lin inp. row

$R(A) = 1$

$\boxed{[1, -1]\begin{bmatrix} b \\ c \end{bmatrix} = t}$

$$\min_{\vec{x}} \|A\vec{x} - \vec{y}\|^2$$

$$\min_{\vec{x}} (A\vec{x} - \vec{y}, A\vec{x} - \vec{y})$$

expands to

$$(A\vec{x}, A\vec{x}) - (\vec{y}, A\vec{x}) - (A\vec{x}, \vec{y}) + (\vec{y}, \vec{y})$$

$\nabla_{x} \Rightarrow 0 \qquad A^T A x = A^T y$

$$\left(\nabla_{\vec{x}} \; (\vec{x}, \vec{x}) \right)_i$$

$$= \frac{\partial}{\partial x_i} \sum_{j=1}^{N} x_j^2 = 2x_i$$

$$\nabla_{\vec{x}} (\vec{x}, \vec{x}) = 2\vec{x}$$

$$\nabla_x \; (A\vec{x}, \vec{y}) = \nabla_x (\vec{x}, A^T y)$$

$$= A^T y$$

go this in components

$$\nabla_{\vec{x}} \; (A\vec{x}, A\vec{x}) = \nabla_{\vec{x}} (x, A^T A x)$$

$$= \nabla_x (A^T A x, x)$$

$$2A^T A x$$