Lecture 24 Shadowitz Chapter 11 sections 2.1 and 2.2

models of college teaching:

-lecture, hmwk, midterm, final (CSM)

-tutor (Oxford and Cambridge)

Questions

-incongruous: How can anyone learn much when education research shows that leaning occurs when students are actively involved?

-congruous: How can I best deliver lectures?

-modifying: How can I make the exams a learning experience rather than a grading mechanism?

-informational: What evidence is there that lectures are effective? What is the goal or objective of this model?

How do you generate a model?

Example: Blaster card to enter Meyer Hall

Questions:

-incongruous: how can it work without a power supply?

-congruous: Faraday's law is the only way power could be transfered. Is this done by the receiver generating an oscillating magnetic field which causes a changing flux in a circuit loop, thereby generating an emf in the blaster card?

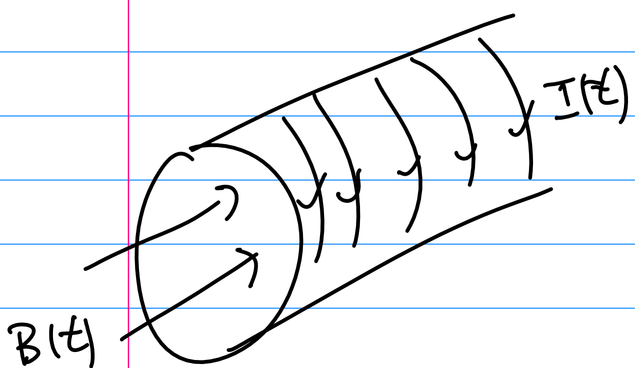
Maxwell's eqns up to now:

$$\vec{\nabla} \cdot \vec{B} = 0 \qquad \vec{\nabla} \times \vec{B} = \mu_0 \vec{J} \qquad \vec{J} = \rho \vec{v}$$

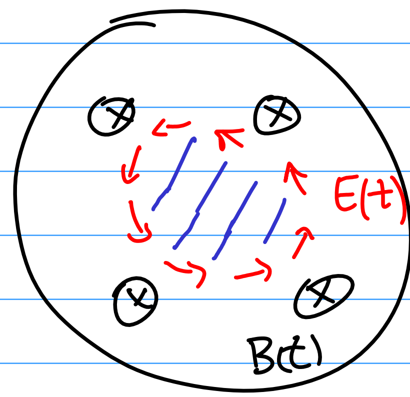
$$\vec{\nabla} \cdot \vec{E} = 0 \qquad \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Stokes theorem

$$\oint \vec{E} \cdot d\vec{r} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{a}$$



side view



end view

Questions:

-analogous: could a changing E generate a B?

-informational: are these equations mathematically consistent?

-informational: how do these appear when written in terms of the vector and scalar potentials?

Are these eqns consistent?

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{V}) = 0$$

How do you prove this?

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{E}) = \vec{\nabla}\cdot\left(-\frac{\partial\vec{B}}{\partial t}\right) = -\frac{\partial}{\partial t}\underbrace{\vec{\nabla}\cdot B}_{=0} = 0 \quad \text{OK}$$

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{B}) = \vec{\nabla}\cdot(\mu_0\vec{J}) = \mu_0\vec{\nabla}\cdot\vec{J} \neq 0$$

But $\vec{\nabla}\cdot\vec{J} = -\frac{\partial\rho}{\partial t} \neq 0$

Questions:

Fix it up by adding another current density whose divergence cancels the one we have.

$$\vec{\nabla}\times\vec{B} = \mu_0(\vec{J} + \vec{J}_{missing})$$

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{B}) = \mu_0\Big(\underbrace{\vec{\nabla}\cdot\vec{J}}_{=-\frac{\partial\rho}{\partial t}} + \underbrace{\vec{\nabla}\cdot\vec{J}_{missing}}_{=\frac{\partial\rho}{\partial t}}\Big) = 0$$

$$\vec{J} = \rho \vec{v}$$

$$\vec{\nabla} \cdot \vec{E} = \rho / \epsilon_0$$

$$\underbrace{\vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t} = -\frac{\partial}{\partial t}\left(\epsilon_0 \vec{\nabla} \cdot \vec{E}\right) = -\vec{\nabla} \cdot \left(\epsilon_0 \frac{\partial \vec{E}}{\partial t}\right)}_{\text{units of } \vec{J}}$$

Let $J_{missing} = \epsilon_0 \frac{\partial \vec{E}}{\partial t}$

$$\vec{\nabla} \cdot \vec{J}_{missing} = \vec{\nabla} \cdot \epsilon_0 \frac{\partial \vec{E}}{\partial t} = \epsilon_0 \frac{\partial}{\partial t} \underbrace{\vec{\nabla} \cdot \vec{E}}_{= \rho / \epsilon_0} = \frac{\partial \rho}{\partial t}$$

$$\vec{J}_{missing} \equiv \vec{J}_{displacement} = \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

Questions:

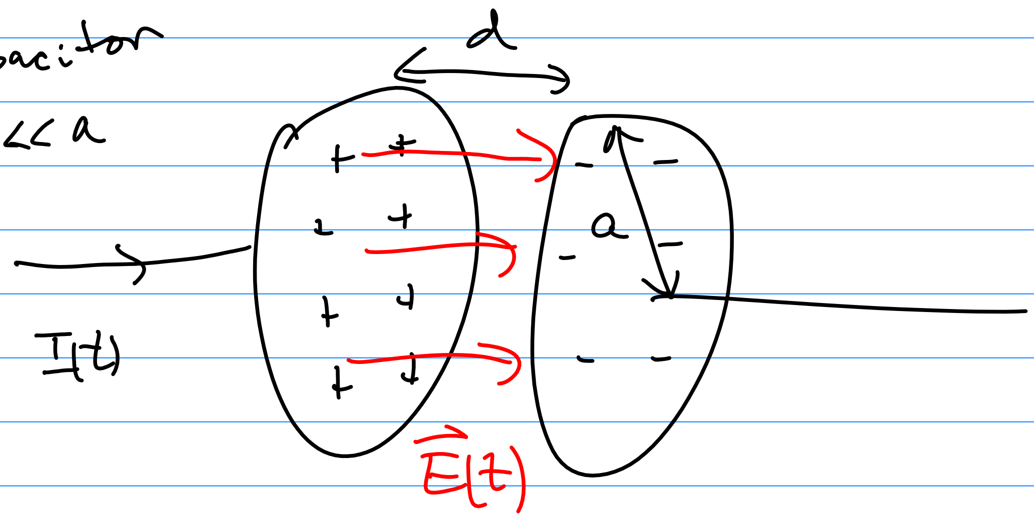
$$\boxed{\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}}$$

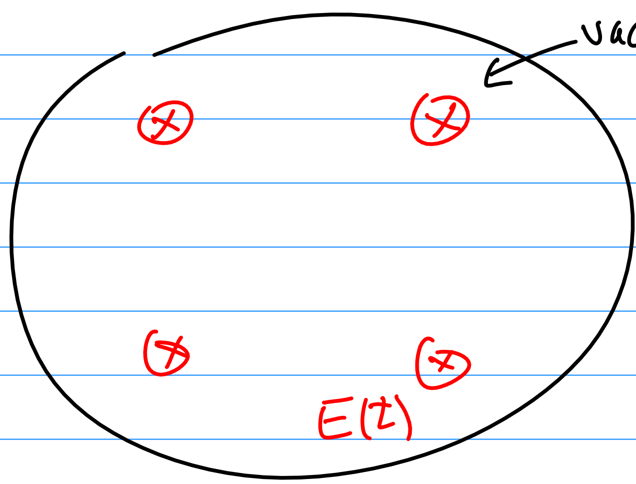
informational: What type of example will illustrate this new law?

informational: How do we choose simplifying assumptions so the example is not full of extraneous effects?

capacitor

$d \ll a$





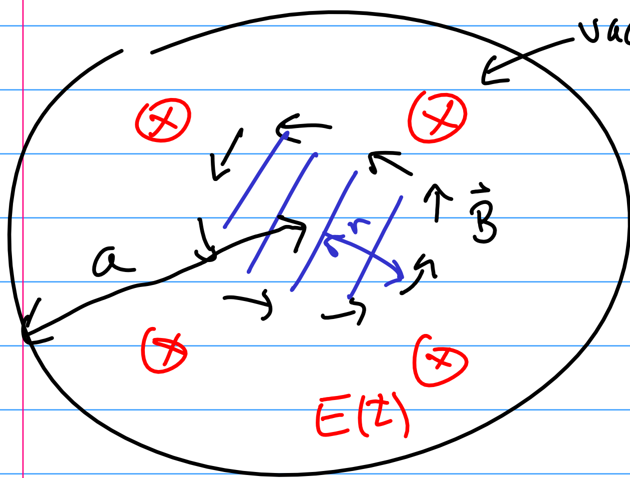
vacuum region between plates

(NO $\vec{J} = \underbrace{\rho}_{=0} \vec{v} = 0$)

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}_{displacement} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

stokes theorem

$$\int \vec{\nabla} \times \vec{B} \cdot d\vec{a} = \oint \vec{B} \cdot d\vec{r} = \mu_0 \int \vec{J}_{disp} \cdot d\vec{a}$$



vacuum region between plates
(NO $\vec{J} = \underset{0}{\underset{\shortparallel}{\rho}} \vec{v} = 0$)

$$\oint \vec{B} \cdot d\vec{r} = \mu_0 \int \epsilon_0 \frac{\partial \vec{E}}{\partial t} \cdot d\vec{a} = \mu_0 \epsilon_0 \frac{\partial}{\partial t} \int \vec{E} \cdot d\vec{a}$$

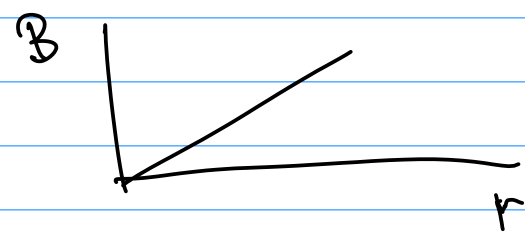
$$B\, 2\pi r = \mu_0 \epsilon_0 \frac{\partial E}{\partial t} \pi r^2$$

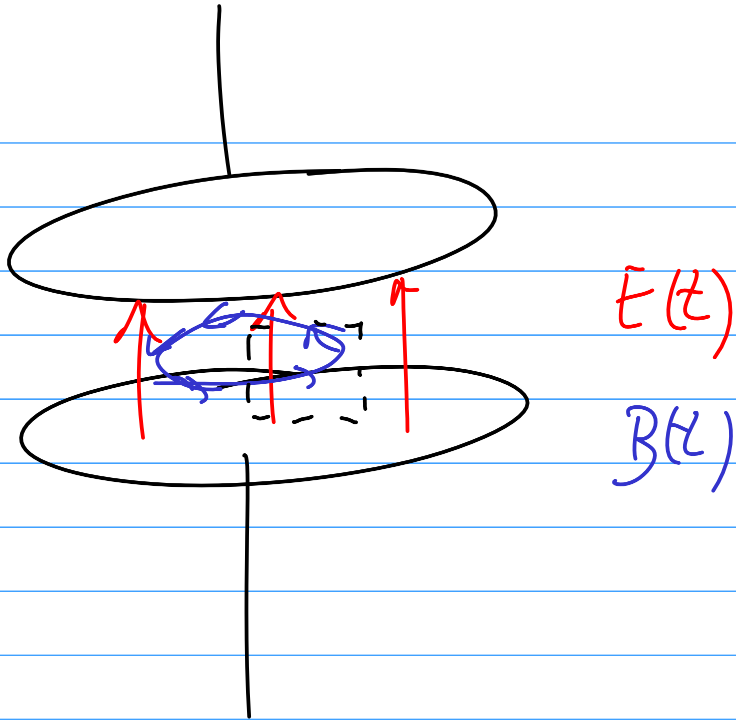
$$E = \frac{\sigma}{\epsilon_0} = \frac{1}{\epsilon_0} \frac{Q}{A}$$

$$\frac{\partial E}{\partial t} = \frac{1}{\epsilon_0} \frac{1}{\pi a^2} \frac{dQ}{dt} = \frac{1}{\epsilon_0} \frac{I}{\pi a^2}$$

$$B\, 2\pi r = \mu_0 \epsilon_0 \pi r^2 \frac{1}{\epsilon_0} \frac{I}{\pi a^2} = \mu_0 \frac{r^2}{a^2} I$$

$$B = \frac{\mu_0}{2\pi} \frac{r}{a^2} I$$





E(t)
B(t)