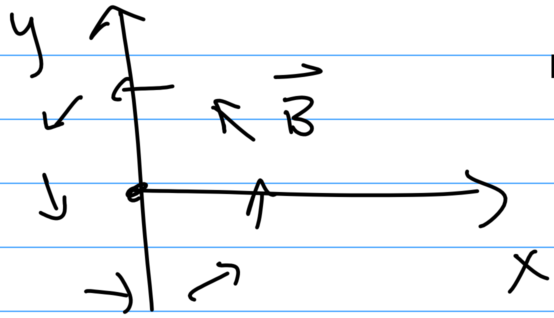
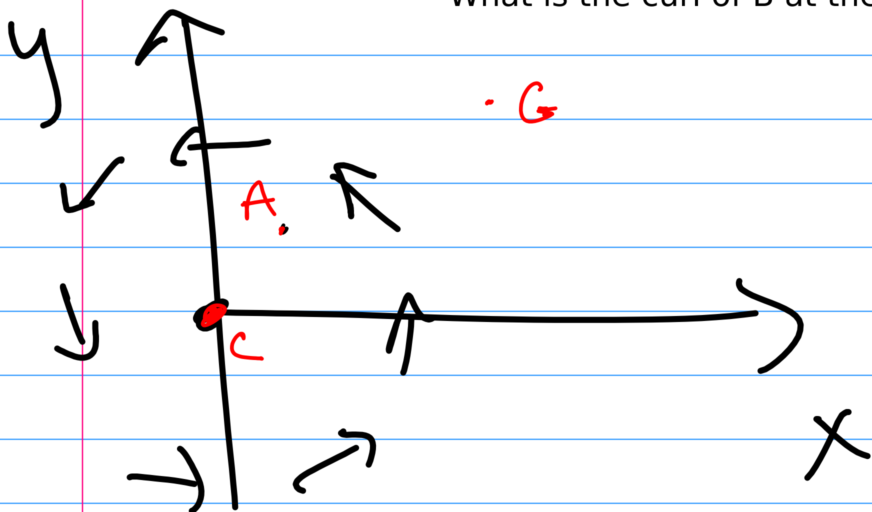
Lecture 9 Shadowitz finish chapter 3 and begin chapter 4.



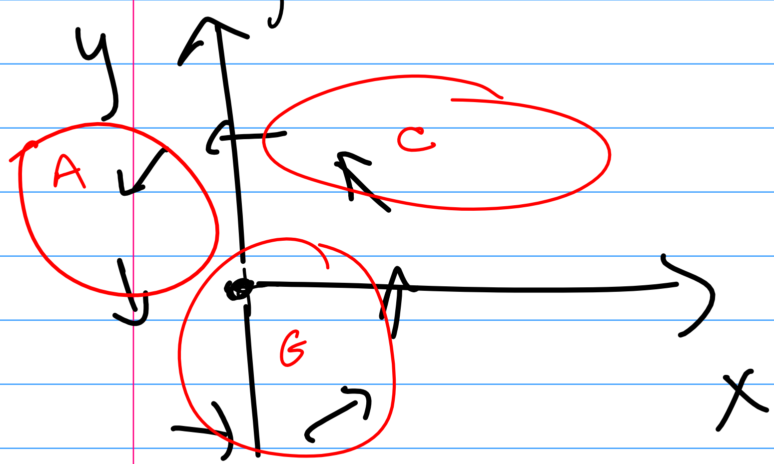
Infinite wire on the z axis carrying current I

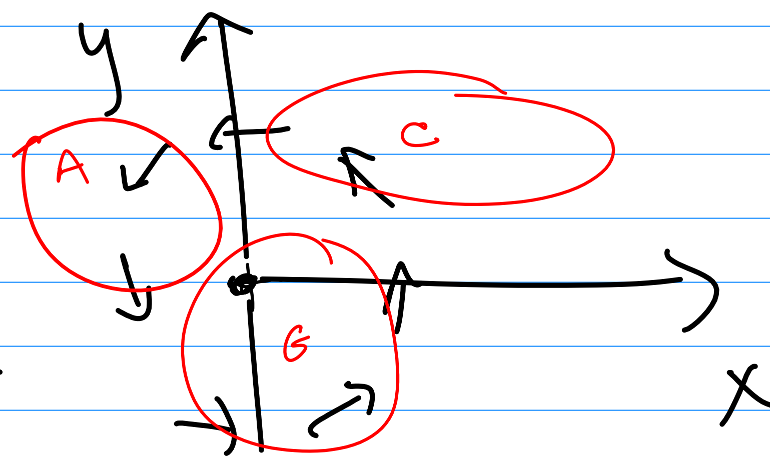
What is the curl of B at these points?



Why did you say that (informational)?

What is the $\int \vec{\nabla} \times \vec{B} \cdot d\vec{a}$ in these surfaces





Why did you say that (informational)? $\int \vec{\nabla} \times \vec{B} \cdot d\vec{a} = \oint \vec{B} \cdot d\vec{r}$

Which of the $\oint \vec{B} \cdot d\vec{r}$ around the perimeter of each surface is non-zero?

We left off here in Monday's lecture:

What is $\vec{\nabla} \cdot B$?

$$\vec{\nabla} \cdot \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}\,') \times (\vec{r} - \vec{r}\,')}{(\vec{r} - \vec{r}\,')^3} dx'dy'dz'$$

$$= \frac{\mu_0}{4\pi} \int \vec{\nabla} \cdot \left[\frac{\vec{J}(\vec{r}\,') \times (\vec{r} - \vec{r}\,')}{(\vec{r} - \vec{r}\,')^3} \right] dx'dy'dz'$$

$$\vec{\nabla} \cdot \left(\frac{\vec{J} \times \hat{\mathscr{r}}}{\mathscr{r}^2} \right) = \frac{\hat{\mathscr{r}}}{\mathscr{r}^2} \cdot (\vec{\nabla} \times \vec{J}) - \vec{J} \cdot \left(\vec{\nabla} \times \frac{\hat{\mathscr{r}}}{\mathscr{r}^2} \right)$$

How do you prove this vector identity (informational)?

$$\vec{\nabla} \cdot (\vec{J} \times \vec{G}) = \vec{\nabla} \cdot \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ J_x & J_y & J_z \\ G_x & G_y & G_z \end{vmatrix} = \vec{\nabla} \cdot \left[\hat{x} (J_y G_z - G_y J_z) - \hat{y} (\right.$$

$$\vec{G} = \frac{\hat{\mathscr{r}}}{\mathscr{r}^2}$$

$$= \frac{\partial}{\partial x} (J_y G_z - G_y J_z) - \frac{\partial}{\partial y} (J_x G_z - G_x J_z) + \frac{\partial}{\partial z} (J_x G_y - G_x J_y)$$

$$\vec{\nabla}\cdot\vec{B} = 0$$

How can the divergence of B be zero if there is a source of B (incongruous)?

Doesn't the divergence tell us about sources of the vector field (incongruous)?

what about $\vec{\nabla}\times\vec{B}$ (causal/creative)?

$$\vec{\nabla}\times\vec{B} = \frac{\mu_0}{4\pi}\int \vec{\nabla}\times\left[\frac{\vec{J}(\vec{r}\,')\times(\vec{r}-\vec{r}\,')}{(\vec{r}-\vec{r}\,')^3}\right] dx'\,dy'\,dz'$$

$$= \mu_0 \vec{J}(x,y,z) \text{ not } \vec{J}(x',y',z')$$

Why is J a function of the unprimed variables(incongruous)?

$$\boxed{\vec{\nabla}\times\vec{B}(x,y,z) = \mu_0\vec{J}(x,y,z)}$$

Apply Stokes theorem

$$\int \vec{\nabla}\times\vec{B}\cdot d\vec{a} = \oint \vec{B}\cdot d\vec{r}$$

$$\boxed{\int \mu_0\vec{J}\cdot da = \oint \vec{B}\cdot d\vec{r} \qquad \text{Ampere's Law}}$$

What have we covered?

1.) Forces on currents

$$d\vec{F} = I\,d\vec{r} \times \vec{B} = \vec{K} \times \vec{B}\,da = \vec{J} \times \vec{B}\,d\text{volume}$$

2.) Finding $\vec{B}$ given current density

$$\vec{B}(x,y,z) = \frac{\mu_0}{4\pi}\int \frac{I\,d\vec{r}\,' \times \hat{\imath}}{\imath^2}$$

$$\vec{B}(x,y,z) = \frac{\mu_0}{4\pi}\int \frac{\vec{K} \times \hat{\imath}}{\imath^2}\,da' \qquad \vec{K}(x',y',z')$$

$$\vec{B}(x,y,z) = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}\,') \times (\vec{r}-\vec{r}\,')}{(\vec{r}-\vec{r}\,')^3}\,dx'dy'dz'$$

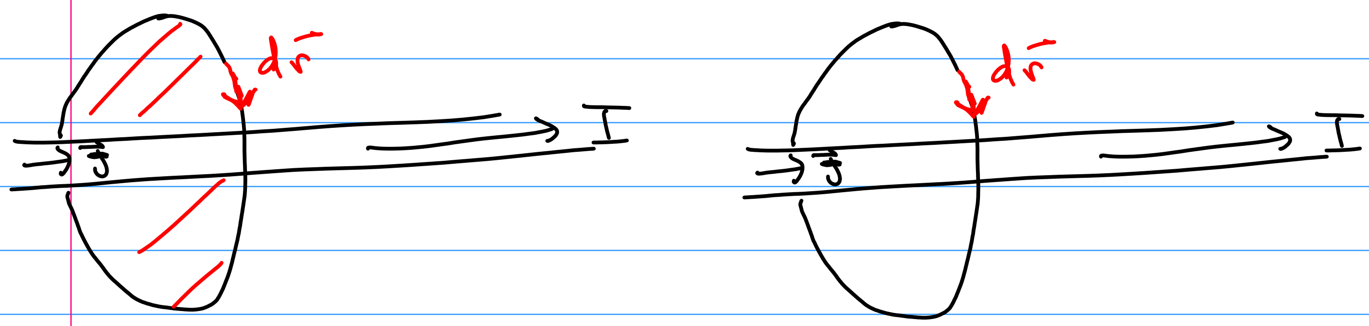
3.) $\vec{\nabla}\cdot\vec{B} = 0 \qquad \vec{\nabla}\times\vec{B} = \mu_0\vec{J}$

$$\int \vec{\nabla}\cdot\vec{B}\cdot d\vec{a} = \int \mu_0\vec{J}\cdot d\vec{a} = \oint \vec{B}\cdot d\vec{r}$$

Helmholtz theorem says we need $\vec{\nabla}\cdot$ & $\vec{\nabla}\times$ vector function to uniquely determine it.

Questions about $\int \mu_0 \vec{J} \cdot d\vec{a} = \oint \vec{B} \cdot d\vec{r}$?

There are infinitely many surfaces that share the same boundary line. Which one are we supposed to use (congruous)?



What are we going to cover?

1) Ampere's law examples

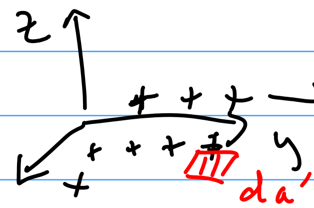
2.) More Biot-Savart examples

3.) Back to electrostatics to find $\vec{\nabla} \times \vec{E} = ?$ $\quad \vec{\nabla} \cdot \vec{E} = \rho/\epsilon_0$

4.) Define voltage and potential energy in electrostatics.

1.) Infinite planar sheet of charge moving at constant speed v and charge density sigma.

How do you calculate B (congruous)?



$\vec{J} = \sigma v_0 \hat{x}$

Method 1

$$\vec{B}(x,y,z) = \frac{\mu_0}{4\pi} \int \frac{\vec{K} \times \hat{\mathscr{r}}}{\mathscr{r}^2} \, da' \; ; \quad \vec{K}(x',y',z') = \sigma v_0 \hat{x}$$

$da' =$

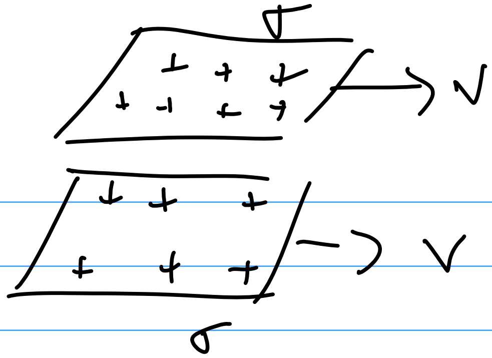
$\vec{r} =$

$\vec{r}\,' =$ $\qquad \vec{K} \times \vec{\mathscr{r}} =$

Method 2 . . . . . . use Ampere's law $\quad \mu_0 \int \vec{J} \cdot d\vec{a} = \oint \vec{B} \cdot d\vec{r}$

What do we know and what do we want to find out (informational)?

2.) Two infinite planar sheets of charge moving.



What do we know and what do we want to find out (informational)?

$\vec{\nabla} \cdot \vec{E} = \rho / \epsilon_0$ $\vec{E} = \int \frac{k\,dQ}{r^2} \hat{r}$

The Helmholtz theorem says we need curl of E along with its divergence to specify E in a finite region of space.

How do I calculate the curl of E (congruous)? $\vec{\nabla} \times \vec{E} = ?$

Questions?

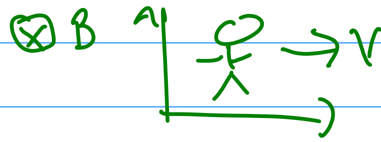
What about the work done in a magnetic field where the curl is non-zero (analogy)?

(2) Fix charges so v = 0 then F = q E + 0

$V \leftarrow$ − − − − − . . . ← Infinite lines of charge

. . + + + + + $\rightarrow V$ $I \rightarrow 2\lambda V$

Q ⊕ → V What is the force on Q?



Go to a reference frame moving to the right at speed V

$2V \leftarrow$ − − − − − . . .

. . + + + + +

Q ⊕ What is the force on Q?

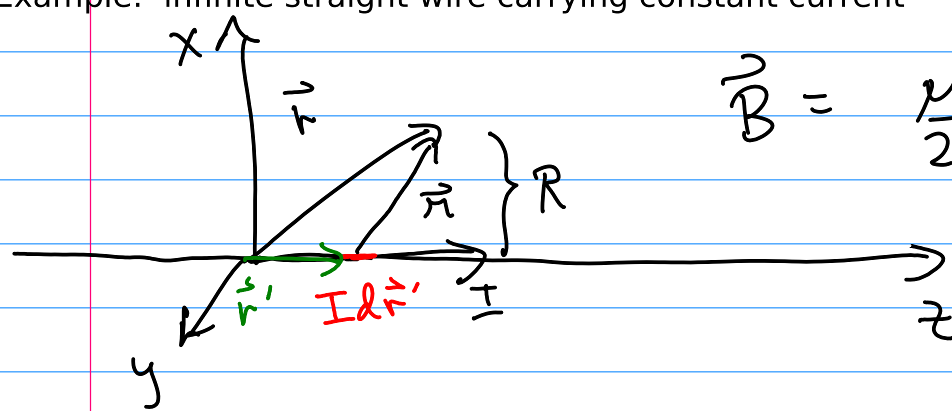
There is NO $q\vec{v}\times\vec{B}$ since $v=0$

$$\vec{F} = q\vec{E} + \underbrace{q\vec{v}\times\vec{B}}_{0 \text{ since } v=0} \quad \text{so} \quad E \neq 0$$

Moving charge density is greater due to length contraction so the negative charge density is greater than the positive thus attracting Q as expected.

We will continue with a mathematica analalysis once we know B from an infinite current moving along a line.

Example: infinite straight wire carrying constant current



$$\vec{B} = \frac{\mu_0}{2\pi R}\hat{\varphi}$$