

Homework 8
Due at the beginning of class March 26

1. You will model the ocean as a conductor to determine if you can find the Titanic using a 'probe' consisting of a conductor suspended below a moving ship and held at a potential of 100 V. Assume that no current can flow anywhere but to the ocean bottom made of rock which is at ground (or 0 Volts) potential. Most of this bottom is covered with thick silt which acts somewhat as an insulator allowing only a small amount of current to flow through it to the rock bottom. The Titanic rests on the rock bottom, not the silt, and therefore is at ground potential. Using the relaxation method solve for the potential at three positions of the probe laterally displaced relative to the Titanic. Explain how you set up the boundary conditions and how you would calculate the total current from the probe using this solution.
2. Complete the details of the derivation of the wave equation, as described in the lecture notes of Mar. 19.
3. Complete the details of the application of Maxwell's equations in both integral and differential form, as described in the lecture notes of Mar. 19, on the plane electromagnetic wave.
4. Complete the details of the perturbative application of Maxwell's equations, as described in the lecture notes of Mar. 19, on the capacitor driven by a harmonic source.
5. Complete the details of the derivation of Poynting's theorem, as described in the lecture notes of Mar. 21.
6. In this problem you will derive an expression for the momentum of a charged particle in a magnetic field that incorporates the vector potential. This is used in the Schrödinger equation to determine how magnetic fields effect the motion of a charged particle. Consider a region either outside or inside a solenoid whose current changes with time. Using the integral form of Faraday's law show that $\vec{E} = -\partial\vec{A}/\partial t$. This electric field then exerts a force $q\vec{E}$ on our charged particle. Show that the change in momentum of the particle is $\Delta m\vec{v} = -q\vec{A}$.
There is at least one thing odd about this. The magnetic field can change very quickly leading to a quick change in \vec{A} . However, the charged particle can have a large mass; be me for instance. Obviously my momentum doesn't change very quickly. The momentum that doesn't change instantly is the dynamical (canonical) momentum $p_{\text{dynamical}} = m\vec{v} + q\vec{A}$. The momentum operator used in quantum mechanics is $\hat{p} = \frac{\hbar}{i}\vec{\nabla} = m\vec{v} + q\vec{A}$ which leads to the momentum term in the kinetic energy being $m\vec{v} = \frac{\hbar}{i}\vec{\nabla} - q\vec{A}$. Ask the internet to look at the form of the Schrödinger equation with both vector and scalar potentials.
7. A plane wave, passing the origin ($x = 0$), is described by $E_0e^{i(-\omega t)}$. Determine the expression for the electric field of this wave after it reflects from a mirror moving at constant speed using the retarded time as described in lecture. At $t = 0$ the mirror is located at $x = L$.
8. MORE PROBLEMS may FOLLOW LATER.