**Homework 5**
**Due at the beginning of class Feb. 19**

1. A charge q is fixed a distance $d$ above an infinite conductor located in the x-y plane. The voltage above the plane is measured to be $\frac{1}{4\pi\epsilon_0}(\frac{q}{\sqrt{x^2+y^2+(z-d)^2}} - \frac{q}{\sqrt{x^2+y^2+(z+d)^2}})$. Derive an expression for the charge density everywhere on the conducting surface.

2. In lecture 27 I started explaining how the force on a test charge moving parallel to a current carrying wire can be explained in terms of relativity. The current is composed of charge with density $\lambda_0$ moving parallel to the test charge and density $-\lambda_0$ moving anti-parallel to the test charge, where $\lambda_0$ is the charge density in the rest frame of the charges (a meter stick with say 1000 charges glued to the meter stick at rest). In this problem you will show how the Lorentz force in one frame has the same parametric dependence as that for a relativistic calculation in the frame of the charge upon which the force is calculated. (a) Express the Lorentz force in the frame where the particle moves at speed $V$ and the positive charge with speed $V$ and negative charge with speed $-V$. (b) In the frame moving with the test charge there is no Lorentz force, only an electric force due to the difference in charge on the wire. Applying length contraction derive an expression to lowest order in $v/c$ the electric force. This force should have the same parametric dependence as in part (a). You will need to use the expression for the speed of light given by $c = 1/\sqrt{\epsilon_0\mu_0}$.

3. Use Excel to numerically solve Laplace's equation using the relaxation method. Look up "relaxation (iterative method)" on wikipedia and read it. Apply the procedure by choosing a 20 by 20 rectangular grid. On the outside perimeter let the voltage be zero. On a 5 by 5 rectangular grid somewhere in the interior let the voltage be 100.

   In the cell where the voltage isn't fixed an equation needs to be inserted. Choose the upper left cell and insert the equation with the syntax equal sign followed by the equation. To have this equation appropriately adjusted for the other cells first outline the cell into which you want the formula to be inserted, then go to edit, choose fill and then fill right (or down). By clicking on a another cell you can check if the formula has been appropriately inserted.

   The spreadsheet automatically iterates through the formulas you create in the cells. However, if the formula is circular (changes values depending upon the previous iteration) then the software spits out an error message. To avoid this you need to turn off the default iteration. This is done by clicking on tools then options then calculations then click on manual then iteration and set the iteration to one. Click on OK. Now to iterate you just press f9. Watch the numbers in the cells change for each iteration. Then graph this array using 3-D plot. Watch how the plot changes for each iteration.

   In OpenOffice go to tools, options, OpenOffice.org calc, calculate, then enable iterations and set iterate step to 1 and minimum change to 100. Press control shift F9 to iterate the calculation.

   Turn in plots and voltage tables of your calculation for each of 3 iterations which illustrate convergence.

4. Charge q is at on corner of a square (side a). At the two neighboring corners are charges -q. (a) How much work does it take to bring in charge q to the last corner from infinity? (b) How much work is required to make this structure?

5. Find the energy stored in a uniformly charged sphere of radius R and charge q by (a) $W = \frac{1}{2}\int \rho V d\tau$ (b) $W = \frac{\epsilon_0}{2}\int_{all\ space} E^2 d\tau$ (c) $W = \frac{\epsilon_0}{2}(\int_{volume} E^2 d\tau + \oint_{surface} V\vec{E}\cdot d\vec{a})$, where $d\tau = dvolume$.

6. Determine the radius of a hydrogen atom using the following simplified model. Find an expression for the electrostatic potential energy of the electron and proton as a function of separation. Find an expression for the kinetic energy as a function of separation where this separation is used in the Heisenberg uncertainty principle to determine the electrons momentum and therefore its kinetic energy. Minimize the total energy to derive an expression for the radius. Check to see how well this model yields the accepted value of the ground state radius.