

Homework 19
Due at the beginning of class Feb. 5

1. A charge q is fixed a distance d above an infinite conductor located in the x - y plane. The voltage above the plane is measured to be $\frac{1}{4\pi\epsilon_0} \left(\frac{q}{\sqrt{x^2+y^2+(z-d)^2}} - \frac{q}{\sqrt{x^2+y^2+(z+d)^2}} \right)$. Derive an expression for the charge density everywhere on the conducting surface.
2. In lecture 27 I started explaining how the force on a test charge moving parallel to a current carrying wire can be explained in terms of relativity. The current is composed of charge with density λ_0 moving parallel to the test charge and density $-\lambda_0$ moving anti-parallel to the test charge, where λ_0 is the charge density in the rest frame of the charges (a meter stick with say 1000 charges glued to the meter stick at rest). In this problem you will show how the Lorentz force in one frame has the same parametric dependence as that for a relativistic calculation in the frame of the charge upon which the force is calculated. (a) Express the Lorentz force in the frame where the particle moves at speed V and the positive charge with speed V and negative charge with speed $-V$. (b) In the frame moving with the test charge there is no Lorentz force, only an electric force due to the difference in charge on the wire. Applying length contraction derive an expression to lowest order in v/c the electric force. This force should have the same parametric dependence as in part (a). You will need to use the expression for the speed of light given by $c = 1/\sqrt{\epsilon_0\mu_0}$.
3. Use Excel to numerically solve Laplace's equation using the relaxation method. Look up "relaxation (iterative method)" on wikipedia and read it. Apply the procedure by choosing a 20 by 20 rectangular grid. On the outside perimeter let the voltage be zero. On a 5 by 5 rectangular grid somewhere in the interior let the voltage be 100.

In the cell where the voltage isn't fixed an equation needs to be inserted. Choose the upper left cell and insert the equation with the syntax equal sign followed by the equation. To have this equation appropriately adjusted for the other cells first outline the cell into which you want the formula to be inserted, then go to edit, choose fill and then fill right (or down). By clicking on a another cell you can check if the formula has been appropriately inserted.

The spreadsheet automatically iterates through the formulas you create in the cells. However, if the formula is circular (changes values depending upon the previous iteration) then the software spits out an error message. To avoid this you need to turn off the default iteration. This is done by clicking on tools then options then calculations then click on manual then iteration and set the iteration to one. Click on OK. Now to iterate you just press f9. Watch the numbers in the cells change for each iteration. Then graph this array using 3-D plot. Watch how the plot changes for each iteration.

In OpenOffice go to tools, options, OpenOffice.org calc, calculate, then enable iterations and set iterate step to 1 and minimum change to 100. Press control shift F9 to iterate the calculation.

Turn in plots and voltage tables of your calculation for each of 3 iterations which illustrate convergence.

4. Charge q is at on corner of a square (side a). At the two neighboring corners are charges $-q$. (a) How much work does it take to bring in charge q to the last corner from infinity? (b) How much work is required to make this structure?
5. Find the energy stored in a uniformly charged sphere of radius R and charge q by (a) $W = \frac{1}{2} \int \rho V d\tau$ (b) $W = \frac{\epsilon_0}{2} \int_{all\ space} E^2 d\tau$ (c) $W = \frac{\epsilon_0}{2} \left(\int_{volume} E^2 d\tau + \oint_{surface} V \vec{E} \cdot d\vec{a} \right)$, where $d\tau = dvolume$.
6. Find the magnetic vector potential of a finite segment of straight wire carrying current I . Put the wire on the z axis from z_1 to z_2 . Do the integral.
7. Find the magnetic field for your previous answer using the curl in the cylindrical system.
8. Find the vector potential above and below a sheet of current moving in the x - y plane along the x axis.
9. For an infinite straight wire carrying constant current I along the z axis, $\vec{A} = A(s)\hat{z}$ where s is the radial coordinate in the cylindrical coordinate system. (a) Find $A(s)$ and show that (b) $\nabla \cdot \vec{A} = 0$ and (c) $\nabla \times \vec{A} = \vec{B}$ using the cylindrical coordinate system.