PHGN 462 Homework 2

1a) Write down the complete Maxwell equations in differential form. Then describe, in words and sentences, what each of them is saying. And by that I mean talk about the physical content – your answer should not be of the form "This one is saying that del dot E equals rho over epsilon naught." If an equation includes multiple kinds of source (like free and bound), indicate them separately.

b) Now write down the Maxwell equations in integral form. Clearly indicate which integrals are over lines, areas, or volumes, and clearly indicate which domains are open and which are closed. Also note that there's more than one acceptable integral form of Faraday's law; use what you like.

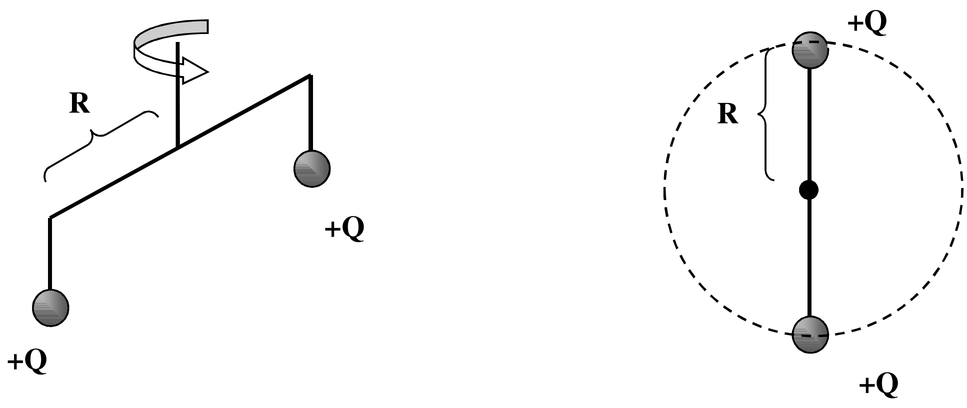
c) Show me how to get from the differential form of Faraday's law to the integral form. If you're restricting the situation somehow, make sure you explain how and why. Then show me how to get from the integral form to the differential form. Explain all the steps.

d) Show me how the Maxwell equations need to change if we want to accommodate magnetic monopoles.* Explain the changes. This information is pretty commonly available, and you should feel free to look it up, but take the time to understand what you're reading and to describe it in your own words.

From this point on, I'm going to assume that you're fluent in the Maxwell equations, so make sure you've got this all down (not so much the magnetic monopole stuff, but the other stuff definitely).

*Note that in recent years people have observed collective magnetic effects that have monopole-like features. However, the real prize, true monopoles that exist as discrete particles, are still nowhere to be found.

2) The picture shows two balls hanging from a crossbar that's free to rotate (like in a mobile, if anyone knows what those are anymore). Each ball has some positive charge Q and mass M. The crossbar is of total length 2R. There's a perspective view (left) and a top-down view (right). The arrow shows one possible rotation; it's free to rotate in either direction.



Now let's suppose the region inside the dotted circle is filled with a uniform magnetic field **B** oriented into the page. That field is turned off linearly over some time interval of length T (that is, B as a function of time is a linear function, and takes T to go from max value to zero). Describe the ensuing motion of the balls, and calculate their final angular momentum

Since we started with no motion and ended with motion, and changed only the fields, we kind of have to conclude that fields, even *static* ones, can have both linear and angular momentum if we want those quantities to be conserved (and just to be clear, we want that very much). We'll be talking about field momentum in more detail soon enough.

3a) Let's start with a straight infinite wire of with some steady current I running through it. Write down the magnetic field it makes. Note that this is an idealized 1-D, zero-radius wire.

b) Now calculate the energy contained in that magnetic field. Or, more precisely, calculate the energy per meter of wire, since it's infinitely long.

c) If you did it right, something should have gone terribly, terribly wrong in part b, owing to the fact that a 1-D wire is not an entirely realistic model of a wire. This happens sometimes. Models work until you push on them too hard, and then they break. Now model the wire as having some finite radius a, as wires generally do, and calculate the new fields and energy per meter. Does that fix it? Comment one way or the other.

d) Now find the energy contained in the electric field of an electron.

e) You probably got something divergent again, if you tried to treat an electron as a point object. The thing is, an actual observed electron (as opposed to the wavefunction describing where it might be) *is* a radius-zero point, as far as we can tell. But there *are* error bars on that zero. Read up on what we know about the size of an electron, and see if you can fix things so that you can get a finite result for the electron's field energy. Go ahead and plug in numbers, too, to make sure this doesn't generate something absurd like 10^{100} Joules or something.

Note: Self-energies are notoriously tricky things. Infinities show up a lot and, frankly, it can be hard to get rid of them. See for example the whole notion of renormalization in quantum field theory.

4) Let's suppose you have a chunk of neutral dielectric material. You heat it up, apply an external electric field to polarize it, and cool it down while the field is in place. Then you take the field away. In this fashion, you can manufacture an object that stays polarized even though there may not be any external fields or charges in the neighborhood (such an object is called an electret and is an actual thing).

Let's further suppose that we have a finished electret that is spherical in shape with radius *a* and has been given a polarization throughout of the form:

$$\vec{P} = kr\hat{r}$$

Where *k* is an arbitrary constant. Note that we haven't added or removed any net charge from it, and the source of the electric field that did the polarizing is long gone. There's nothing present except the polarized sphere.

a) Find the bound volume charge density ρ_b in this sphere and the bound surface charge σ_b. Explicitly calculate the total bound volume charge and the total bound surface charge and compare them. Is the result sensible?

b) Calculate the electric field **E** inside and outside the sphere. Then calculate the electric displacement field **D** inside and outside the sphere.

c) Give me a sketch and a qualitative description of how the charges in the sphere must be arranged to produce what we have here.