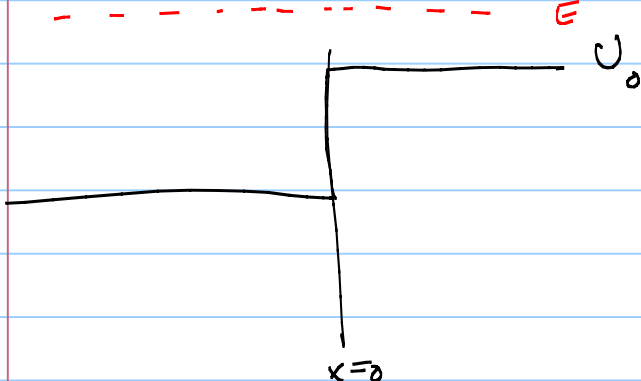


exam 2 sample questions : solutions

Note Title

2/27/2008

- 1) A particle is incident from left with energy $E > U_0$. Compute the reflection and transmission coeff.



$$x < 0 \quad \psi'' = -\frac{2mE}{\hbar^2} \psi \Rightarrow \psi'' + k^2 \psi = 0$$
$$\sqrt{2mE}/\hbar = k$$

$$\psi(x) = A e^{ikx} + B e^{-ikx}$$

$$x > 0 \quad \psi'' = -\frac{2m}{\hbar^2} (E - U_0) \psi \Rightarrow \psi'' + \ell^2 \psi = 0$$

$$\psi = F e^{i\ell x} \quad \ell = \frac{\sqrt{2m(E - U_0)}}{\hbar}$$

cont. of ψ : $A + B = F$

ψ' : $i\hbar(A - B) = i\ell F$

$$\Rightarrow A + B = \frac{\ell}{k} (A - B)$$

$$B(1 + \ell/k) = (\frac{\ell}{k} - 1)A$$

$$\underline{B/A = \frac{k}{e} - 1 / (1 + k/e)} \Rightarrow \boxed{\left(\frac{k-e}{e+k}\right)^2 = R}$$

$$B = F - A, \quad \frac{-\cancel{k}eF + \cancel{k}eA}{\cancel{k}e} = B$$

$$F - A = A - \frac{e}{k} F \Rightarrow F \left(1 + \frac{e}{k}\right) = 2A$$

$$\Rightarrow F(k+e) = 2Ae$$

$$\Rightarrow \frac{F}{A} = \frac{2e}{k+e}$$

$$T + R = 1$$

$$\Rightarrow \boxed{T = \frac{4k^2}{(k+e)^2}}$$

$$\frac{(k-e)^2}{(e+k)^2} + \frac{4k^2}{(e+k)^2} = \frac{k^2 + 2ke + e^2}{(e+k)^2} = \frac{(k+e)^2}{(k+e)^2} = 1$$

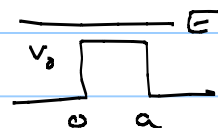
2) See the Scan on the wiki within the questions

3) See 3/3/08 lecture notes for setup.

I) $Ae^{ik_1x} + Be^{-ik_1x}$

II) $Ce^{ik_2x} + De^{-ik_2x}$

III) $F e^{ik_1x}$



cont. of ψ : a) $A + B = C + D$
 b) $C e^{ik_2a} + D e^{-ik_2a} = F e^{ik_1a}$

ψ' c) $ik_1(A - B) = ik_2(C - D)$

d) $ik_2(C e^{ik_2a} - D e^{-ik_2a}) = ik_1 F e^{ik_1a}$

a) $\Rightarrow A + B - C = D$

c) $\Rightarrow -\frac{k_1}{k_2}(A - B) + C = D$

$\Rightarrow D = A + B - \frac{k_1}{k_2}(A - B)$

$C = \frac{1}{2} \left[A + B + \frac{k_1}{k_2}(A - B) \right]$

Now use these to replace C & D in

b) + d)

$$\frac{k_2}{k_1} \left[c e^{ik_2 a} - D e^{-ik_2 a} \right] = c e^{ik_2 a} - D e^{-ik_2 a}$$

$$D = A + B - \frac{k_1}{k_2} (A - B) = A \left[1 - \frac{k_1}{k_2} \right] + B \left[1 + \frac{k_1}{k_2} \right]$$

$$C = \frac{1}{2} \left[A + B + \frac{k_1}{k_2} (A - B) \right] = \frac{1}{2} A \left[1 + \frac{k_1}{k_2} \right] + \frac{1}{2} B \left[1 - \frac{k_1}{k_2} \right]$$

$$\Rightarrow \left(\frac{k_2}{k_1} - 1 \right) c e^{ik_2 a} = \left(\frac{k_2}{k_1} + 1 \right) D e^{-ik_2 a}$$

$$\left(\frac{k_2}{k_1} - 1 \right) e^{ik_2 a} \left[\frac{1}{2} A \left[1 + \frac{k_1}{k_2} \right] + \frac{1}{2} B \left[1 - \frac{k_1}{k_2} \right] \right]$$

$$= \left(\frac{k_2}{k_1} + 1 \right) e^{-ik_2 a} \left[A \left[1 - \frac{k_1}{k_2} \right] + B \left[1 + \frac{k_1}{k_2} \right] \right]$$

$$(k_2 - k_1) e^{2ik_2 a} \left[\frac{1}{2} A \left[1 + \frac{k_1}{k_2} \right] + \frac{1}{2} B \left[1 - \frac{k_1}{k_2} \right] \right]$$

$$= (k_2 + k_1) \left[A \left[1 - \frac{k_1}{k_2} \right] + B \left[1 + \frac{k_1}{k_2} \right] \right]$$

$$A \left\{ (k_2 - k_1) e^{2ik_2 a} \frac{1}{2} \left(1 + \frac{k_1}{k_2} \right) - (k_2 + k_1) \left(1 - \frac{k_1}{k_2} \right) \right\}$$

$$= B \left\{ -(k_2 - k_1) e^{2ik_2 a} \frac{1}{2} \left(1 - \frac{k_1}{k_2} \right) + (k_1 + k_2) \left(1 + \frac{k_1}{k_2} \right) \right\}$$

$$\Rightarrow A \left\{ \frac{e^{2ik_2 a}}{2} (k_2 - k_1) (k_2 + k_1) - (k_2 + k_1) (k_2 - k_1) \right\}$$

$$= B \left\{ -\frac{e^{2ik_2 a}}{2} (k_2 - k_1) (k_2 - k_1) + (k_1 + k_2) (k_2 + k_1) \right\}$$

This allows you to write B/A .
Similar argument for A/A .

$$4) \quad E_N = \alpha N^2$$

$$E_{N+1} = \alpha (N+1)^2 \Rightarrow E_{N+1} - E_N = \alpha (N^2 + 2N + 1) - \alpha N^2$$

$$\Rightarrow \frac{E_{N+1} - E_N}{E_N} = \frac{2N+1}{N^2}$$

$$5) \quad a^+ = \sqrt{\frac{m\omega}{2\hbar}} \left(x - \frac{i\hbar}{m\omega} p \right) \\ = \sqrt{\frac{\alpha}{2}} \left(x - \frac{i}{m\omega} \left(-i\hbar \frac{d}{dx} \right) \right)$$

$$\psi_0(x) = \left(\frac{\alpha}{\pi} \right)^{1/4} e^{-y^2/2} \quad y = \sqrt{\alpha} x$$

apply a^+ to ψ_0 and normalize to get

$$a^+ \psi_0 = \left(\frac{\alpha}{\pi} \right)^{1/4} \sqrt{2} y e^{-y^2/2}$$

plug this into Schrodinger with
the potential $V(x) = \frac{1}{2} m \omega^2 x^2$
and you will get $E = \frac{3}{2} \hbar \omega$.

$$6) S = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix}$$

$$S^{\dagger} S = \begin{pmatrix} S_{11}^* & S_{21}^* \\ S_{12}^* & S_{22}^* \end{pmatrix} \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} = \begin{pmatrix} |S_{11}|^2 + |S_{21}|^2 & S_{11}^* S_{12} + S_{21}^* S_{22} \\ S_{12}^* S_{11} + S_{22}^* S_{21} & |S_{12}|^2 + |S_{22}|^2 \end{pmatrix}$$

$$S^{\dagger} S = I \Rightarrow 1) |S_{11}|^2 + |S_{21}|^2 = 1 = T_e + R_e$$

$$2) |S_{12}|^2 + |S_{22}|^2 = 1 = T_r + R_r$$

$$3) S_{12}^* S_{11} + S_{22}^* S_{21} = 0$$

$$4) S_{11} S_{12} + S_{21} S_{22} = 0$$

1) + 2) are energy conservation

$$a) \quad \psi(x) = \alpha x e^{-\beta x} e^{i\omega t/\hbar}$$

$$\psi'(x) = \alpha e^{-\beta x} - \alpha \beta x e^{-\beta x}$$

$$\psi''(x) = -\alpha \beta e^{-\beta x} - \alpha \beta e^{-\beta x} + \alpha \beta^2 x e^{-\beta x}$$

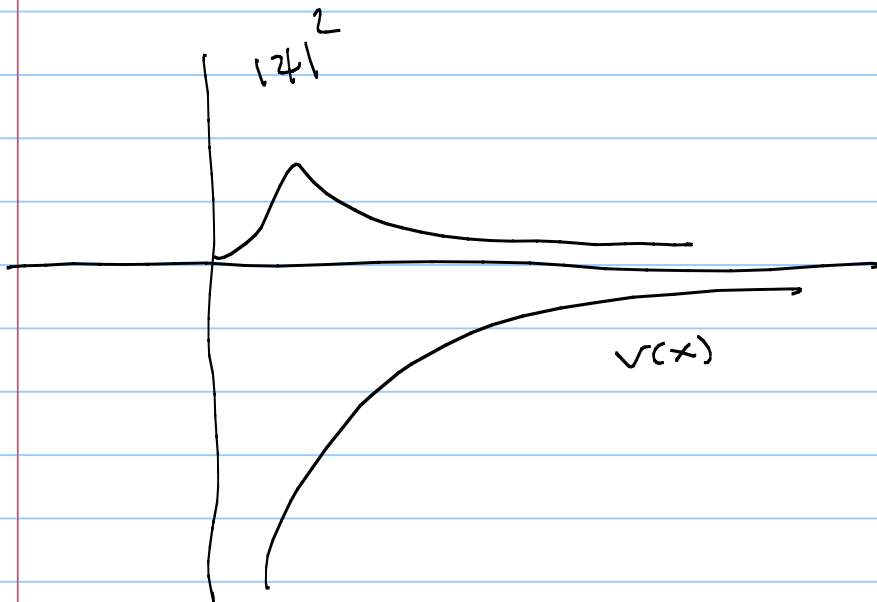
$$= -2\alpha \beta e^{-\beta x} + \alpha \beta^2 x e^{-\beta x}$$

$$\Rightarrow -\psi''(x) = \left(\frac{2\beta}{x} - \beta^2 \right) \alpha x e^{-\beta x}$$

Schrödinger ($\hbar^2/2m = 1$)

$$-\psi'' = (E - V_0) \psi$$

So if we put $V(x) = -\frac{2\beta}{x}$, $E = -\beta^2$



Note Title

3/3/2008

The page contains a series of horizontal blue lines for writing, starting from the top header area and extending down to the bottom of the page. A vertical red line is positioned on the left side, creating a margin. The lines are evenly spaced and cover the majority of the page's width.