

Forced, damped simple harmonic osc.

$$\ddot{x} + \gamma \dot{x} + \omega_0^2 x = F/m \quad F(t)$$

$\gamma \equiv$ damping coefficient $[\gamma] = \frac{1}{\text{time}}$

$$\omega_0^2 = \frac{k}{m}$$

Simplify by letting $F(t) \rightarrow F_0 e^{i\omega t}$

See later: if we can solve for $F = e^{i\omega t}$ then we can solve for any $F(t)$ **Fourier Series**

$$X(t) = X_0 e^{i\omega t}$$

$$X_0 = \underbrace{F_0/m}$$

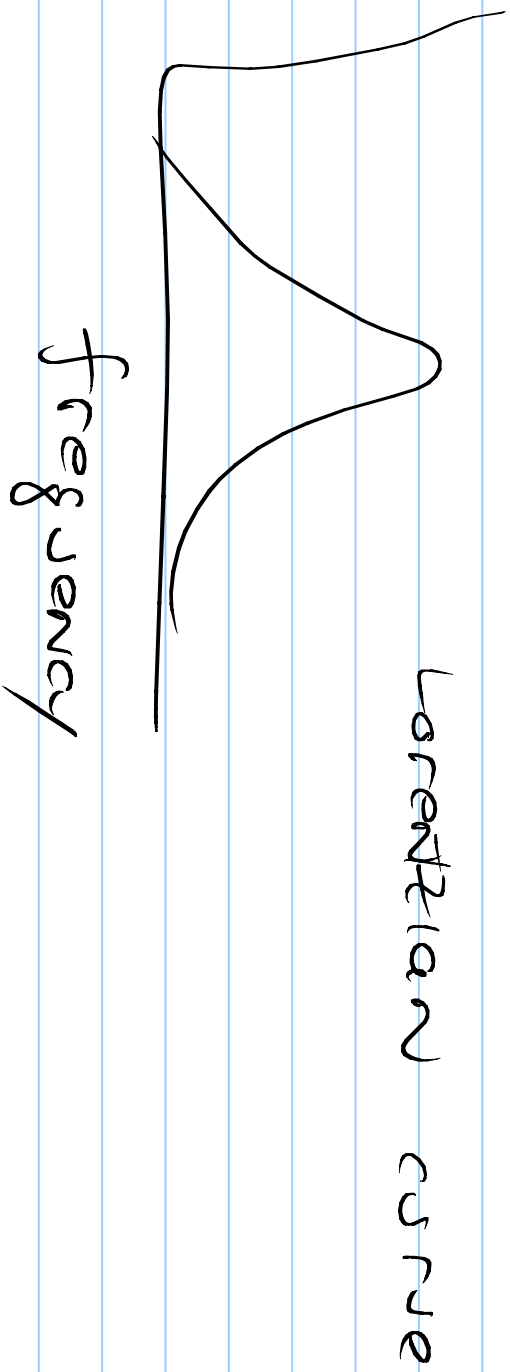
$$\underbrace{[(\omega_s^2 - \omega^2) + i\gamma\omega]}$$

$$\sqrt{(\omega_s^2 - \omega^2)^2 + \gamma^2 \omega^2} \quad \text{mag. of denom.}$$

Phase ϕ of denom $\tan^{-1} \left[\frac{\gamma\omega}{\omega_s^2 - \omega^2} \right]$

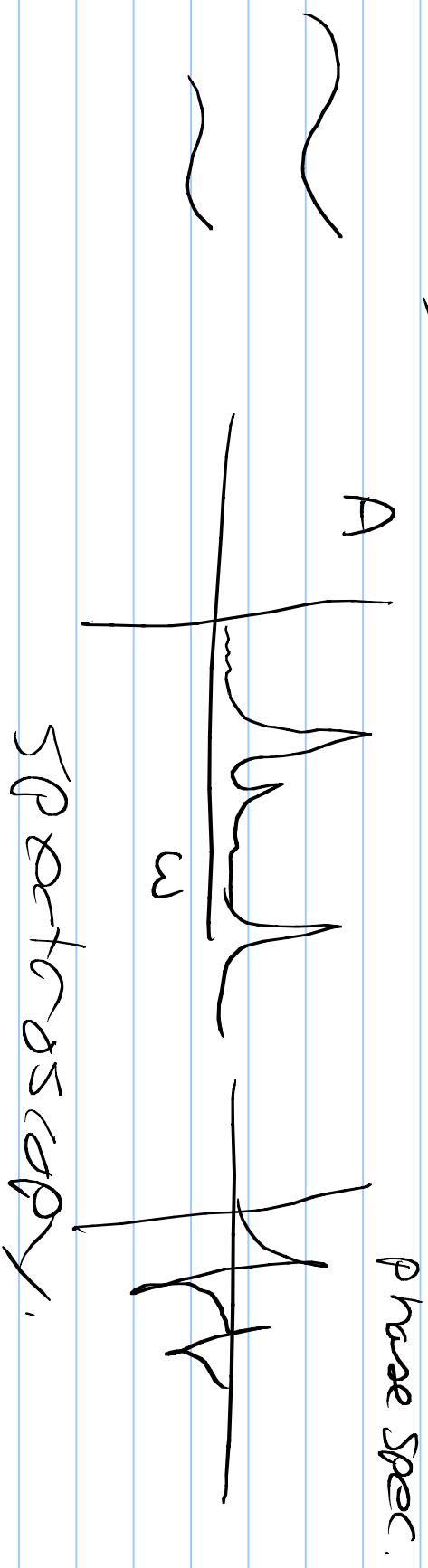
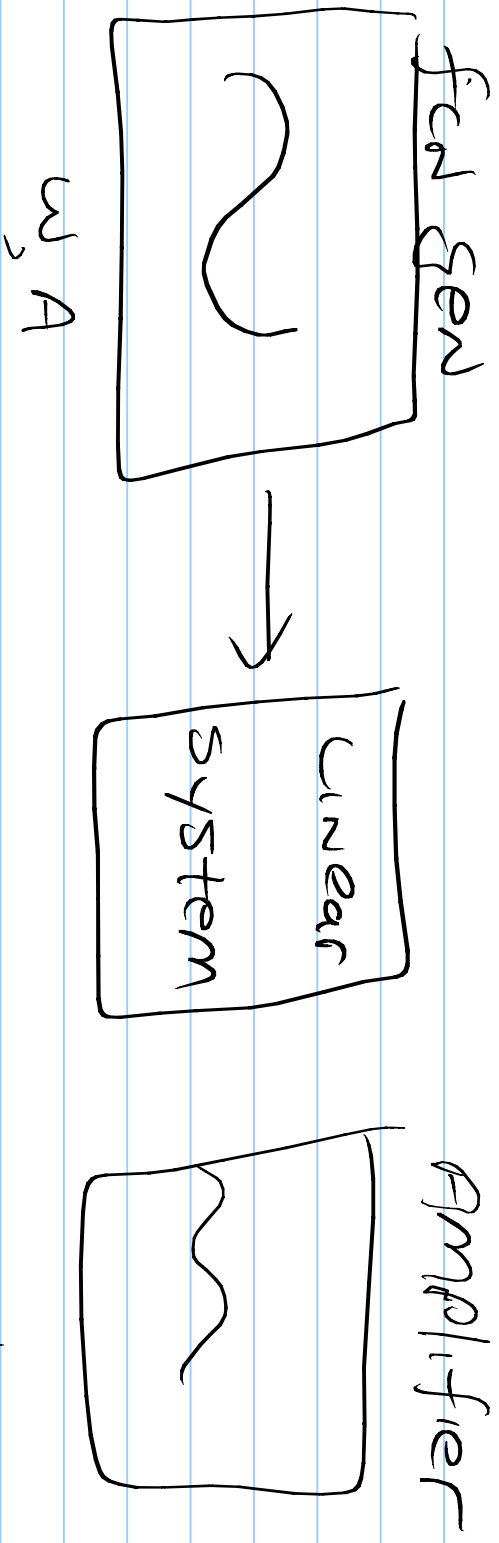
$$X(\omega) = e^{i\omega t} \left[\frac{1}{\sqrt{(\omega_0^2 - \omega^2)^2 + \gamma^2 \omega^2}} e^{-i\gamma \omega} \left[\frac{\gamma \omega}{\omega_0^2 - \omega^2} \right] \right]$$

AMP.



Amplitude Spectrum

Phase Spectrum



Spectroscopy.

iX

Euler

$$(e^{iX})^2 = e^{i2X} = \cos 2X + i \sin 2X$$

$$(e^{iX})^n = \cos nX + i \sin nX$$

$-iX$

$$e^{iX} + e^{-iX} = 2 \cos X$$

$$\cos X = \frac{e^{iX} + e^{-iX}}{2}$$

$$e^{ix} - e^{-ix} = 2i \sin x$$

$$\sin x = \frac{e^{ix} - e^{-ix}}{2i} \quad \text{inverse Euler}$$

$$e^{iy} = \cos y + i \sin y$$

suppose $x = iy$ y real

$$e^{-y} = \cos(iy) + i \sin(iy)$$

$X = iy$ Y real

$$\frac{e^{ix} + e^{-ix}}{2} = \cos x$$

$$\frac{e^{ix} - e^{-ix}}{2i} = \sin x$$

$x \rightarrow iy$

$$\frac{e^{-y} + e^y}{2} = \cos(iy)$$

$$\frac{e^{-y} - e^y}{2i} = \sin iy$$

$$\frac{e^y + e^{-y}}{2} = \cosh(y) \quad i \frac{e^y - e^{-y}}{2} = \sinh(y)$$

$$\cos(iy) = \cosh(y) \quad -i \sin(iy) = \sinh(y)$$