2.3.4 Use set builder notation to specify the following sets:

(a) The set of all integers greater than or equal to 5.
(b) The set of all even integers.
(c) The set of all positive rational numbers.
(d) The set of all real numbers greater than 1 and less than 7.

2.3.5 For each of the following sets, use English to describe the set or use the roster method to specify all of the elements of the set.

(a) $\{x \in \mathbb{R} \mid -3 \leq x \leq 5\}$
(b) $\{x \in \mathbb{Z} \mid -3 \leq x \leq 5\}$
(c) $\{x \in \mathbb{R} \mid x^2 = 16\}$

2.4.3 Assume the universal set for each variable is the set of integers. Write each of the following statements as an English sentence that does not use the symbols for quantifiers.

(a) $(\exists m)(\exists n)(m > n)$
(b) $(\exists m)(\forall n)(m > n)$
(c) $(\forall m)(\exists n)(m > n)$
(d) $(\forall m)(\forall n)(m > n)$
(e) $(\exists n)(\forall m)(m^2 > n)$
(f) $(\forall n)(\exists m)(m^2 > n)$

3.1.8 Determine if each of the following propositions is true or false. Justify each conclusion.

(a) For all integers a and b, if $ab \equiv 0 \pmod 6$, then $a \equiv 0 \pmod 6$ or $b \equiv 0 \pmod 6$.
(b) For all $a \in \mathbb{Z}$, if $a^2 \equiv 4 \pmod 8$, then $a \equiv 2 \pmod 8$.

3.1.11 Let r be a positive real number. The equation for a circle of a radius r whose center is the origin is $x^2 + y^2 = 1$.

(a) Use implicit differentiation to determine $\dfrac{dy}{dx}$
(b) Let (a, b) be a point on the circle with $a \neq 0$ and $b \neq 0$. Determine the slope of the line tangent to the circle at the point (a, b).
(c) Prove that the radius of the circle to the point (a, b) is perpendicular to the line tangent to the circle at the point (a, b).

3.1.12 Determine if each of the following statements is true or false. Provide a counterexample for statements that are false and provide a complete proof for those that are true.

(a) For all real numbers x and y, $\sqrt{xy} \leq \dfrac{x+y}{2}$.
(b) For all real numbers x and y, $\sqrt{xy} \leq \left(\dfrac{x+y}{2}\right)^2$.
(c) For all nonnegative real numbers x and y, $\sqrt{xy} \leq \dfrac{x+y}{2}$.

3.1.13 Use one of the true inequalities in Exercise (12) to prove the following proposition.

For each real number, a, the value of x that gives the maximum value of $x(a-x)$ is $x = \dfrac{a}{2}$.

3.2.10 Prove that for each integer a, if $a^2 - 1$ is even, then 4 divides $a^2 - 1$.

3.2.11 Prove that for all integers a and m, if a and m are the lengths of the sides of a right triangle and $m + 1$ is the length of the hypotenuse, then a is an odd integer.

3.3.16 Three natural numbers a, b, and c with $a < b < c$ are called a **Pythagorean triple** provided that $a^2 + b^2 = c^2$. For example, the numbers 3, 4, and 5 form a Pythagorean triple, and the numbers 5, 12, and 13 form a Pythagorean triple.

(a) Verify that if $a = 20$, $b = 21$, and $c = 29$, then $a^2 + b^2 = c^2$, and hence, 20, 21, and 29 form a Pythagorean triple.

(b) Determine two other Pythagorean triples. That is, find integers a, b and c such that $a^2 + b^2 = c^2$.

(c) Is the following proposition true or false? Justify your conclusion.

Let a, b, and c be integers. If $a^2 + b^2 = c^2$, then a is even or b is even.