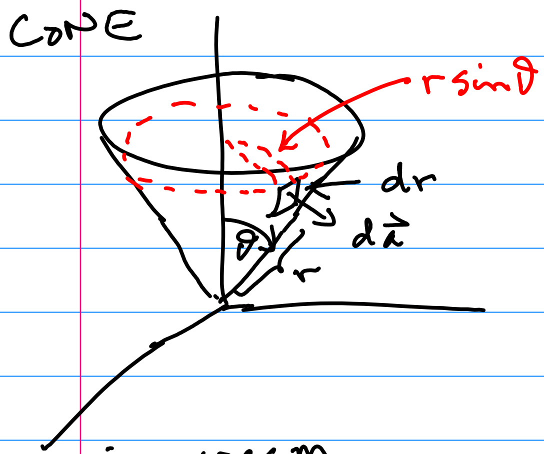
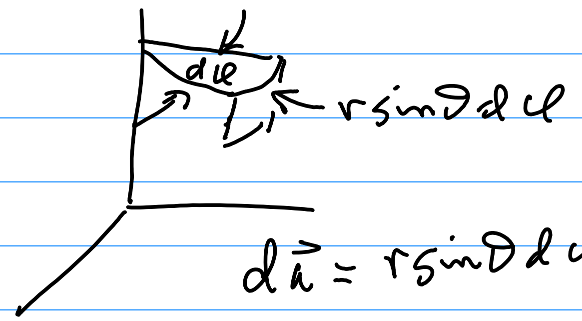
Lecture 9 Shadowitz parts of sections 1-5, 1-6, 1-7, and 3-3 may be covered.

InkSurvey question for Monday

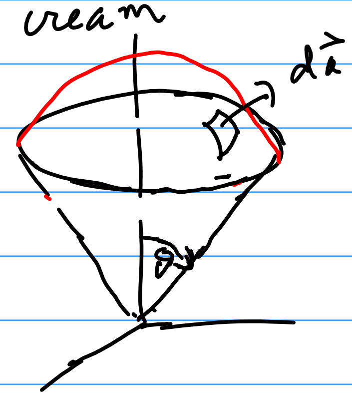
CONE





$$d\vec{a} = r\sin\theta\, d\varphi\, dr\, \hat{\theta}$$

ice cream



$$d\vec{a} = r\sin\theta\, d\varphi\, r\, d\theta\, \hat{r}$$

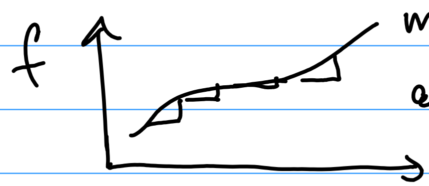
Using the spherical unit vectors is useful when calculating the flux given E in spherical coordinates. Then you have E dot da where the unit vectors go away due to the dot product.

InkSurvey question for Monday

Fundamental theorem of calculus

$$\int_a^b \frac{df}{dx}\,dx = \int_a^b df = f(b) - f(a)$$

$$\lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x}$$



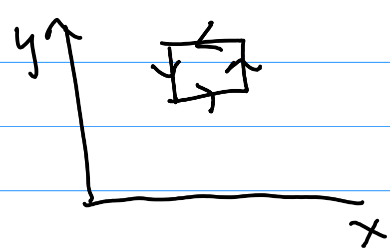
multiply $\frac{\Delta f}{\Delta x}$ by Δx and sum

Defn of curl:

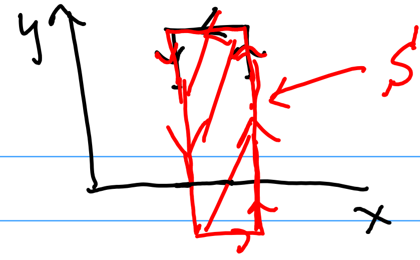
$$(\vec{\nabla}\times\vec{F})_z \equiv \lim_{\Delta S_z \to 0}\left[\frac{1}{\Delta S_z}\oint_{C_z} \vec{F}\cdot d\vec{r}\right]$$

multiply both sides by ΔS_z

$$\vec{\nabla}\times\vec{F}\cdot\vec{\Delta S} = \oint_{\text{around } \Delta S} \vec{F}\cdot d\vec{r}$$



$$\sum_{S'} \vec{\nabla}\times\vec{F}\cdot\Delta\vec{s} = \oint_{\text{around } S'} \vec{F}\cdot d\vec{r}$$

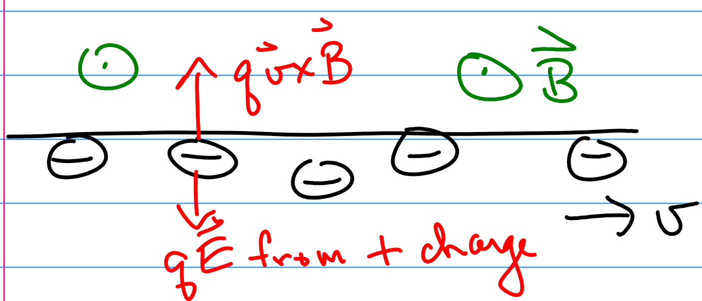


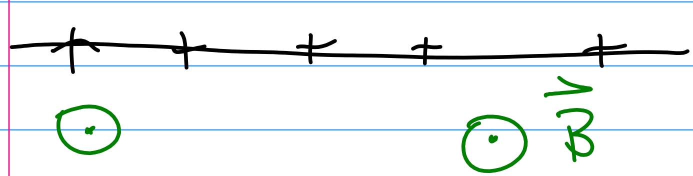
$\Sigma \rightarrow \int \Rightarrow$ Stokes theorem $\oint \vec{\nabla}\times\vec{F}\cdot d\vec{a} = \oint \vec{F}\cdot d\vec{r}$

sum inside bndry — boundary integral

InkSurvey question from Friday

What is the net force on a positive charge?





Muddiest points:

(1) How was the Lorentz force determined?

(2) Cross product in spherical coords.

(3) Interpretation of curl on the surface of water.

(4) Force of wire on itself due to the B it generates.

(5) What's the point of the curl?

(6) So much material. How do I study for the next exam?

(7) Relativity and electric and magnetic fields.

(2) Fix charges so v = 0 then F = q E + 0

Infinite lines of charge

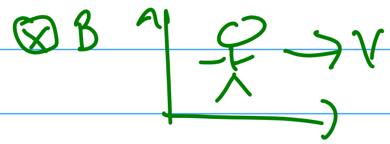
$V \leftarrow$ − − − − − . . .

. . + + + + + $\rightarrow V$

$I \rightarrow 2\lambda V$

Q $\oplus \rightarrow V$

What is the force on Q?



Go to a reference frame moving to the right at speed V

$2V \leftarrow$ − − − − − . . .

. . + + + + +

Q $\oplus$

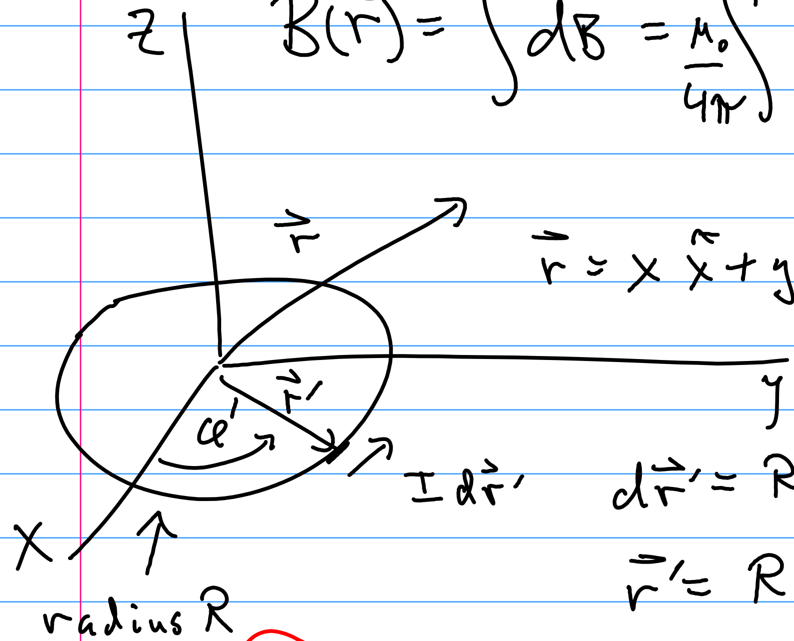
What is the force on Q?

There is NO $q\vec{v}\times\vec{B}$ since $v = 0$

What have we covered?

Calculate B given currents — Law of Biot & Savart

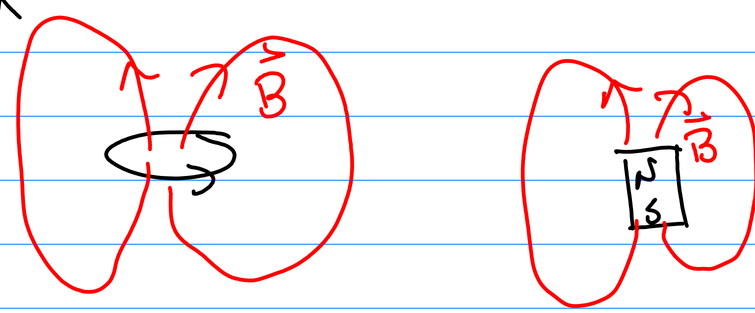
$$\vec{B}(\vec{r}) = \int d\vec{B} = \frac{\mu_0}{4\pi}\int \frac{I\,d\vec{r}\,' \times (\vec{r}-\vec{r}\,')}{(\vec{r}-\vec{r}\,')^3}$$



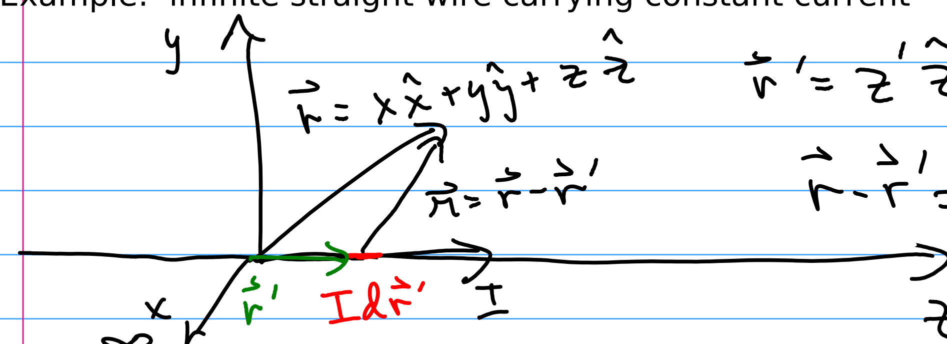
$$\vec{r} = x\,\hat{x} + y\,\hat{y} + z\,\hat{z}$$

$$d\vec{r}\,' = R\,d\varphi'(-\sin\varphi'\,\hat{x} + \cos\varphi'\,\hat{y})$$

$$\vec{r}\,' = R\cos\varphi'\,\hat{x} + R\sin\varphi'\,\hat{y}$$



Example: infinite straight wire carrying constant current



$$\vec{r} = x\,\hat{x} + y\,\hat{y} + z\,\hat{z} \qquad \vec{r}\,' = z'\,\hat{z} \qquad d\vec{r}\,' = dz'\,\hat{z}$$

$$\vec{\mathscr{r}} = \vec{r} - \vec{r}\,'$$

$$\vec{r} - \vec{r}\,' = x\,\hat{x} + y\,\hat{y} + (z - z')\,\hat{z}$$

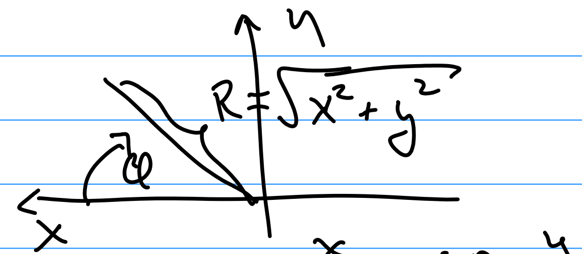
$$\vec{B} = \frac{\mu_0}{4\pi}\int_{-\infty}^{\infty} \frac{I\,d\vec{r}\,' \times (\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3} \quad \vdots \quad \frac{\vec{r}-\vec{r}\,'}{|\vec{r}-\vec{r}\,'|^3} = \frac{x\,\hat{x} + y\,\hat{y} + (z-z')\,\hat{z}}{\left(x^2 + y^2 + (z-z')^2\right)^{3/2}}$$

$$\vec{B} = \frac{\mu_0 I}{4\pi}\int_{-\infty}^{\infty} \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 0 & dz' \\ \frac{x}{(\ )^{3/2}} & \frac{y}{(\ )^{3/2}} & \frac{z-z'}{(\ )^{3/2}} \end{vmatrix}$$

$$\vec{B} = \frac{\mu_0 I}{4\pi}\int\left[\hat{x}\,\frac{dz'\,y}{(\quad)^{3/2}} - \hat{y}\,\frac{dz'\,x}{(\quad)^{3/2}}\right]$$

$$\vec{B} = \frac{\mu_0 I}{4\pi}\hat{x}\int_{-\infty}^{\infty}\frac{dz'\,y}{\left(x^2+y^2+(z-z')^2\right)^{3/2}} - \frac{\mu_0 I}{4\pi}\hat{y}\int_{-\infty}^{\infty}\frac{dz'\,x}{\left(x^2+y^2+(z-z')^2\right)^{3/2}}$$

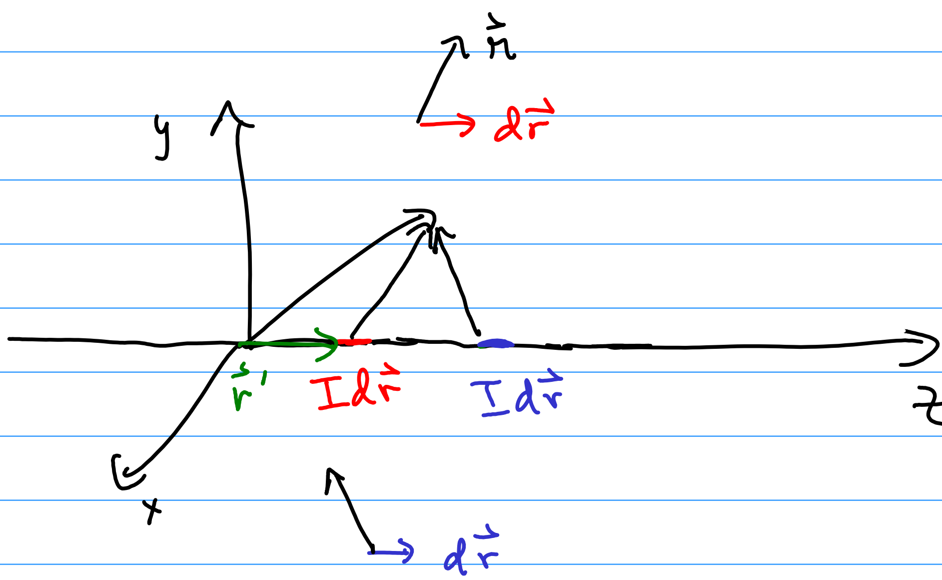
$$\vec{B} = \frac{\mu_0 I}{4\pi}\left(\frac{2y}{x^2+y^2}\hat{x} - \frac{2x}{x^2+y^2}\hat{y}\right)$$



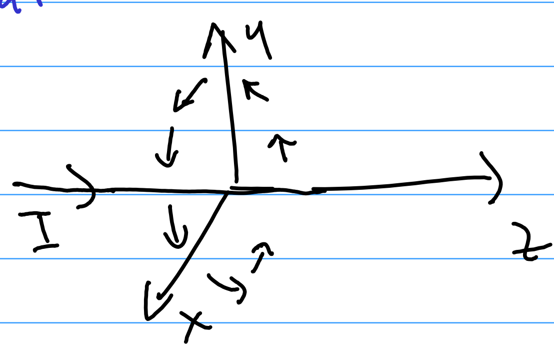
$$\vec{B} = \frac{\mu_0}{2\pi}\left(\frac{1}{R}\sin\varphi\,\hat{x} - \frac{1}{R}\cos\varphi\,\hat{y}\right)$$

$\cos\varphi = \frac{x}{R}$ $\quad \sin\varphi = \frac{y}{R}$

$$\vec{B} = -\frac{\mu_0}{2\pi R}\hat{\varphi}$$



$$\vec{B} = \frac{\mu_0 I}{2\pi r}\hat{\varphi}$$



What are we going to cover?

Helmholtz theorem says we need $\vec{\nabla}\cdot$ & $\vec{\nabla}\times$ vector function to uniquely determine it.

General result

$$\vec{B}(x,y,z) = \int d\vec{B} = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}\,')\times(\vec{r}-\vec{r}\,')}{(\vec{r}-\vec{r}\,')^3}\,dx'dy'dz'$$

What is $\vec{\nabla}\cdot\vec{B}$?

$$\vec{\nabla}\cdot\frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}\,')\times(\vec{r}-\vec{r}\,')}{(\vec{r}-\vec{r}\,')^3}\,dx'dy'dz'$$