Physics 200: Fundamental Equations

Maxwell's Equations

Gauss's Law for Electric Fields: $\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0} = 4\pi k Q_{encl}$

Gauss's Law for Magnetic Fields: $\oint \vec{B} \cdot d\vec{A} = 0$

Electric Flux: $\Phi_E = \int \vec{E} \cdot d\vec{A}$;   Magnetic Flux: $\Phi_B = \int \vec{B} \cdot d\vec{A}$

Ampère/Maxwell: $\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{encl} + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$

Faraday's Law: $\varepsilon_{ind} = \oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$

Fields, Forces and Energy

Electric Field: $d\vec{E} = \frac{kdQ}{r^2}\hat{r} = \frac{kdQ}{r^3}\vec{r}$; $\vec{F}_E = q\vec{E}_{at\ q}$

Electric Potential (Voltage): $V_a - V_b = \int_a^b \vec{E} \cdot d\vec{\ell}$; $E_x = -\frac{dV}{dx}$; $dV = \frac{kdQ}{r}$

Electrostatic Energy: $U_{of\ q} = qV$

Dielectrics: $\epsilon = \kappa_E \epsilon_0$

Magnetic Field: $d\vec{B} = \frac{\mu_0 I d\vec{\ell} \times \hat{r}}{4\pi r^2} = \frac{\mu_0 I d\vec{\ell} \times \vec{r}}{4\pi r^3}$

Magnetic Force: $d\vec{F} = I d\vec{\ell} \times \vec{B}$; $\vec{F}_B = q\vec{v} \times \vec{B}$

Magnetic Dipole: $\vec{\mu} = NI\vec{A}$; $\vec{\tau} = \vec{\mu} \times \vec{B}$

Circuits

Resistors: $dR = \frac{\rho dL}{A}$; $R_{series} = \sum_i R_i$; $R_{parallel} = \left(\sum_i R_i^{-1}\right)^{-1}$

Capacitors: $C = \frac{Q}{V}$; $U_C = \frac{1}{2}CV^2$; $C_{series} = \left(\sum_i C_i^{-1}\right)^{-1}$; $C_{parallel} = \sum_i C_i$; $C = \kappa_E C_0$

Ohm's Law: $V = IR$

Current: $I = \frac{dQ}{dt} = n|q|v_d A$

Power: $P = IV$

Kirchoff's Laws: $\sum_{loop} V_i = 0$; $\sum I_{in} = \sum I_{out}$

RC & LR Circuits: Charging and Discharging equations take the form of $e^{-t/\tau}$ and $1 - e^{-t/\tau}$

$\tau_{RC} = RC$ $\tau_{LR} = \frac{L}{R}$

AC Circuits: $X_C = \frac{1}{\omega C}$; $V_C = IX_C$; $Z = \sqrt{R^2 + X_C^2}$; $V = IZ$; $V_{rms} = \frac{V_{peak}}{\sqrt{2}}$

Inductors: $\varepsilon_{ind} = -L\frac{dI}{dt}$; $L = \frac{N\Phi_{B,1\ turn}}{I}$; $U_L = \frac{1}{2}LI^2$

Inductance: $M_{12} = \frac{N_2 \Phi_{B,1\ turn\ of\ 2}}{I_{in\ 1}}$; $\varepsilon_1 = -M_{12}\frac{dI_2}{dt}$

Electromagnetic Waves, Optics and Field Energy Density

Field Energy Density: $u_E = \frac{1}{2}\epsilon_0 E^2$; $u_B = \frac{1}{2\mu_0}B^2$

Momentum: $p = \frac{U}{c}$

Wave Properties: $v = \lambda f$; $k = \frac{2\pi}{\lambda}$; $\omega = 2\pi f$; $B = \frac{E}{c}$

Intensity: $I = c\frac{1}{2}\epsilon_0 E_m^2 = \frac{P_{avg}}{A}$

Reflection/Refraction: $c_1 = \frac{c}{n_1}$; $\theta_{in} = \theta_{out}$;   Snell-Descartes Law: $n_1 \sin\theta_1 = n_2 \sin\theta_2$

Additional Information/Useful Constants

Common Electric Fields: $E_{inf\ sheet} = \frac{\sigma}{2\epsilon_0}$; $E_{inf\ line} = \frac{2k\lambda}{r}$; $E_{charged\ ring} = \frac{kQx}{(x^2+a^2)^{3/2}}$; $C_{parallel\ plate} = \frac{\epsilon_0 A}{d}$

Common Magnetic Fields: $B_{inf\ wire} = \frac{\mu_0 I}{2\pi r}$; $B_{solenoid} = \mu_0 nI$; $L_{solenoid} = \mu_0 n^2 A\ell$; $B_{current\ loop} = \frac{\mu_0 NIR^2}{2(x^2+R^2)^{3/2}}$

Fundamental Charge: $e = 1.602 \times 10^{-19}$C;   Electron Mass: $m_e = 9.109 \times 10^{-31}$kg

Proton Mass: $m_p = 1.673 \times 10^{-27}$kg

$k = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2}$; $\epsilon_0 = 8.854 \times 10^{-12}\frac{\text{F}}{\text{m}}$

$\mu_0 = 4\pi \times 10^{-7} \approx 12.566 \times 10^{-7}\frac{\text{Tm}}{\text{A}}$

$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3 \times 10^8 \frac{\text{m}}{\text{s}}$

Vector Formulas

$$\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C} = \mathbf{C} \cdot (\mathbf{A} \times \mathbf{B}) = (\mathbf{C} \times \mathbf{A}) \cdot \mathbf{B} = \mathbf{B} \cdot (\mathbf{C} \times \mathbf{A})$$

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B} (\mathbf{A} \cdot \mathbf{C}) - \mathbf{C} (\mathbf{A} \cdot \mathbf{B})$$

$$(\mathbf{A} \times \mathbf{B}) \cdot (\mathbf{C} \times \mathbf{D}) = (\mathbf{A} \cdot \mathbf{C})(\mathbf{B} \cdot \mathbf{D}) - (\mathbf{A} \cdot \mathbf{D})(\mathbf{B} \cdot \mathbf{C})$$

Derivatives of Sums

$$\nabla (f + g) = \nabla f + \nabla g$$

$$\nabla \cdot (\mathbf{A} + \mathbf{B}) = \nabla \cdot \mathbf{A} + \nabla \cdot \mathbf{B}$$

$$\nabla \times (\mathbf{A} + \mathbf{B}) = \nabla \times \mathbf{A} + \nabla \times \mathbf{B}$$

Derivatives of Products

$$\nabla (fg) = f\nabla g + g\nabla f$$

$$\nabla (\mathbf{A} \cdot \mathbf{B}) = \mathbf{A} \times (\nabla \times \mathbf{B}) + \mathbf{B} \times (\nabla \times \mathbf{A}) + (\mathbf{A} \cdot \nabla) \mathbf{B} + (\mathbf{B} \cdot \nabla) \mathbf{A}$$

$$\nabla \cdot (f\mathbf{A}) = f (\nabla \cdot \mathbf{A}) + \mathbf{A} \cdot (\nabla f)$$

$$\nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{A}) - \mathbf{A} \cdot (\nabla \times \mathbf{B})$$

$$\nabla \times (f\mathbf{A}) = f (\nabla \times \mathbf{A}) - \mathbf{A} \times (\nabla f)$$

$$\nabla \times (\mathbf{A} \times \mathbf{B}) = \mathbf{A} (\nabla \cdot \mathbf{B}) - \mathbf{B} (\nabla \cdot \mathbf{A}) + (\mathbf{B} \cdot \nabla) \mathbf{A} - (\mathbf{A} \cdot \nabla) \mathbf{B}$$

Second Derivatives

$$\nabla \times (\nabla \times \mathbf{A}) = \nabla (\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$$

$$\nabla \cdot (\nabla \times \mathbf{A}) = 0$$

$$\nabla \times (\nabla f) = 0$$

Integral Theorems

$$\int_V (\nabla \cdot \mathbf{A}) \, dV = \oint_S \mathbf{A} \cdot \hat{\mathbf{n}} \, dS \qquad \text{Gauss's (divergence) Theorem}$$

$$\int_S (\nabla \times \mathbf{A}) \cdot \hat{\mathbf{n}} \, dS = \oint_C \mathbf{A} \cdot d\boldsymbol{\ell} \qquad \text{Stokes's (curl) Theorem}$$

$$\int_{\mathbf{a}}^{\mathbf{b}} (\nabla f) \cdot d\boldsymbol{\ell} = f(\mathbf{b}) - f(\mathbf{a})$$

$$\int_V \left(f\nabla^2 g - g\nabla^2 f \right) dV = \oint_S (f\nabla g - g\nabla f) \cdot \hat{\mathbf{n}} \, dS \qquad \text{Green's Theorem}$$

Chapter 5, 6, 7, 8 useful relationships

Separation of variables general solutions

Cartesian: $V(x, y, z)$ as combinations of $cos(kx) + sin(kx)$ or $cosh(kx) + sinh(kx)$
or $e^{kx} + e^{-kx}$

Spherical: $V(r, \theta) = \sum_{l=0}^{\infty}(A_l r^l + B_l r^{-l-1})P_l(cos\ (\theta))$

Cylindrical: $V(r, \varphi) = A ln(r) + B + \sum_{n=0}^{\infty}(A_n r^n + B_n r^{-n})(C_n \cos(n\varphi) + D_n \sin(n\varphi))$

Legendre polynomials: $P_0(x) = 1$; $P_1(x) = x$; $P_2(x) = \frac{3}{2}x^2 - 1$; $P_3(x) = \frac{5}{2}x^3 - \frac{3}{2}x$

Legendre orthogonality relationship: $\int_{-1}^{1} P_n(x)P_l(x)\ dx = \delta_{nl}\frac{2}{2n+1}$

Polarization relationships

$\vec{p} = \alpha\vec{E}$ $\vec{P} = \chi_e\varepsilon_0\vec{E}$ $\varepsilon = \varepsilon_0(1 + \chi_e)$ $\kappa = 1 + \chi_e = \frac{\varepsilon}{\varepsilon_0}$

$\sigma_b = \hat{n} \cdot \vec{P}$ $\rho_b = -\vec{\nabla} \cdot \vec{P}$

$\vec{P} = n\vec{p}$ $\alpha = \frac{3\varepsilon_0}{n}\frac{\kappa-1}{\kappa+2}$

$\vec{D} = \varepsilon_0\vec{E} + \vec{P} = \varepsilon\vec{E}$ $\vec{\nabla} \cdot \vec{D} = \rho_f$

Current and current densities

$I = \frac{dQ}{dt} = qn_L v$ $dI = \vec{J} \cdot \vec{dA}$ $\vec{J} = qn\vec{v}$ $\vec{\nabla} \cdot \vec{J} = -\frac{\partial\rho}{\partial t}$

$\vec{J} = \sigma\vec{E}$ $dI = \vec{K} \cdot \widehat{e_\perp}dl$ $RC = \frac{\varepsilon}{\sigma}$

Magnetic vector potential

$\vec{B} = \nabla\times\vec{A}$ $\vec{A}(\vec{x}) = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{x'})}{|\vec{x}-\vec{x'}|}d^3x$

Chapter 9, 10, 11 useful relationships

Magnetization relationships

$$\vec{M} = \chi_m \vec{H} \quad \mu = \mu_0(1 + \chi_m)$$

$$\vec{J}_b = \nabla \times \vec{M} \quad \vec{K}_b = \vec{M} \times \hat{n}$$

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M} = \frac{\vec{B}}{\mu}$$

Forms of Faraday's Law

$$EMF = -\frac{d\Phi}{dt} \qquad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \oint \vec{E} \cdot \vec{dl} = -\frac{d}{dt} \int \vec{B} \cdot \vec{dA}$$

$$\oint \vec{E} \cdot \vec{dl} = -\int \frac{\partial \vec{B}}{\partial t} \cdot \vec{dA}$$

Momentum & energy

$$\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0} \qquad u_{em} = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2\mu_0}B^2 \qquad \frac{d\vec{P}}{dV} = \mu_0 \varepsilon_0 \vec{S}$$

Boundary conditions in matter

$$D_{2,perp} - D_{1,perp} = \sigma_f \qquad\qquad B_{1,perp} = B_{2,perp}$$

$$\vec{E}_{1,parallel} = \vec{E}_{2,parallel} \qquad \vec{H}_{2,parallel} - \vec{H}_{1,parallel} = \vec{K}_f \times \hat{n}$$

Potentials & Gauges

$$\vec{E} = -\nabla V - \frac{\partial \vec{A}}{\partial t} \qquad\qquad \vec{B} = \nabla \times \vec{A} \qquad\qquad Coulomb: \qquad \nabla \cdot \vec{A} = 0$$

$$Transforms: \ V \to V - \frac{\partial f}{\partial t} \qquad \vec{A} \to \vec{A} + \nabla f \qquad\qquad Lorentz: \qquad \nabla \cdot \vec{A} = -\varepsilon_0 \mu_0 \frac{\partial V}{\partial t}$$

Chapter 13, 14, 15 useful relationships

Optics

Brewster's angle: $\tan\theta_B = \frac{n_2}{n_1}$ $\qquad\qquad$ $T = \frac{I_{trans}}{I_{incident}}$ $\qquad$ $R = \frac{I_{reflect}}{I_{incident}}$

Phase velocity $\ v_{ph} = \frac{\omega}{k}$ $\qquad$ Group velocity $\ v_g = \frac{d\omega}{dk}$

Retarded potentials and radiation

Lorentz gauge potentials: $-\nabla^2 V + \frac{1}{c^2}\frac{\partial^2 V}{\partial t^2} = \frac{\rho}{\varepsilon_0}$ $\qquad$ $-\nabla^2 \vec{A} + \frac{1}{c^2}\frac{\partial^2 \vec{A}}{\partial t^2} = \mu_0 \vec{J}$

Larmor formula: $P = \frac{1}{4\pi\varepsilon_0}\frac{2q^2 a^2}{3c^3}$ $\quad$ Scattering cross-section: $\sigma = \frac{P_{avg}}{S_{inc}}$

Lienard-Wiechert: $V(\vec{x},t) = \frac{q}{4\pi\varepsilon_0 R}\frac{1}{1-\hat{n}\cdot\vec{\beta}}$ $\qquad$ $\vec{A}(\vec{x},t) = \frac{q\vec{v}}{4\pi\varepsilon_0 c^2 R}\frac{1}{1-\hat{n}\cdot\vec{\beta}}$

Handbook goodies

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$ $\qquad\qquad$ $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$ $\qquad\qquad$ $e^x = 1 + x + \frac{x^2}{2!} + \cdots$

$\int \frac{du}{(a+bu^2)^{3/2}} = \frac{u}{a\sqrt{a+bu^2}}$ $\qquad\qquad$ $\frac{d}{dx}\left(\frac{1}{\tan x}\right) = \frac{1}{\sin^2 x}$