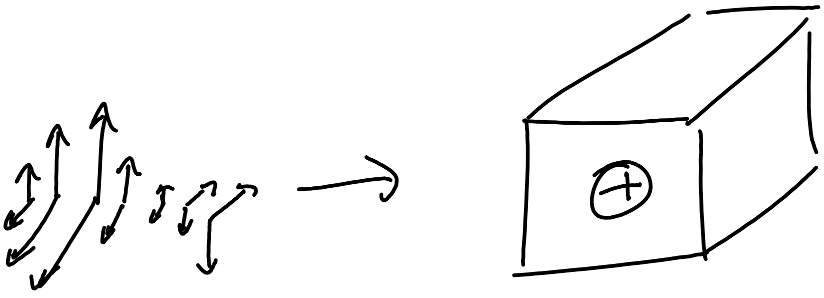
Lecture 28 Shadowitz chapter 6

Review of the Poynting theorem:

surface over which to apply Poyntings th.



$$\frac{dW_{EM}}{dt} = -\frac{d}{dt}\int \underbrace{\frac{1}{2}\left(\epsilon_0 E^2 + \frac{B^2}{\mu_0}\right)}_{u_{EM}} d\tau - \frac{1}{\mu_0}\int \vec{\nabla}\cdot\vec{S}\, d\tau$$

$$W_{net} = \Delta KE$$

$$W_{non\,con} + W_{cons} = \Delta KE$$

$W_{non\,con} = 0$ for a free charge in the box

$W_{cons} = W_{EM} = \Delta KE \Rightarrow$

$$\frac{dW_{EM}}{dt} = \frac{d\Delta KE}{dt} = \frac{d}{dt}\int u_{KE}\, d\tau$$

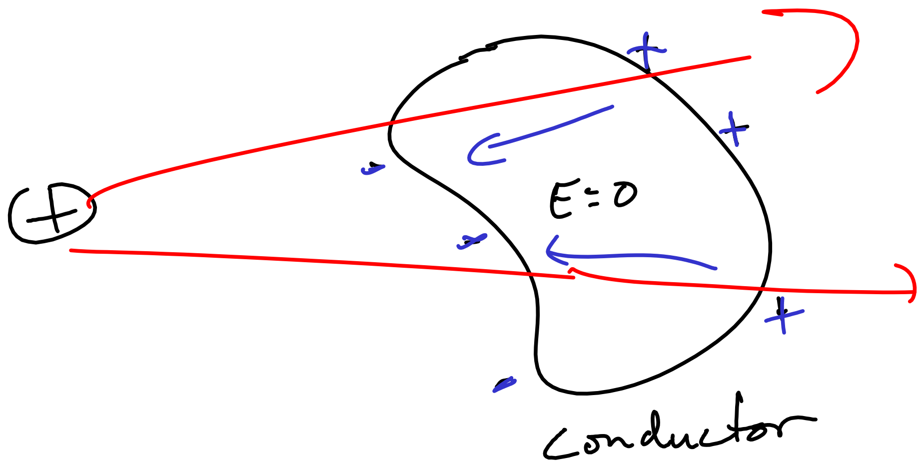
u_{KE} ← KE density

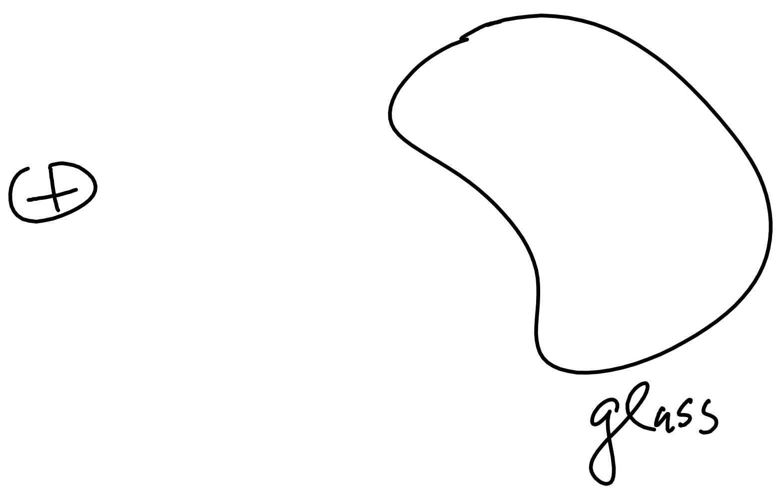
$$-\frac{d}{dt}\int u_{EM}\, d\tau - \frac{1}{\mu_0}\int \vec{\nabla}\cdot\vec{S}\, d\tau = \frac{d}{dt}\int u_{KE}\, d\tau$$

$$\boxed{-\frac{\partial}{\partial t}(u_{EM} + u_{KE}) = \vec{\nabla}\cdot\vec{S}} \qquad \boxed{\vec{\nabla}\cdot\vec{J} = -\frac{\partial \rho}{\partial t}}$$

This conservation law has NO source term! Energy is conserved with no source or sink of energy.

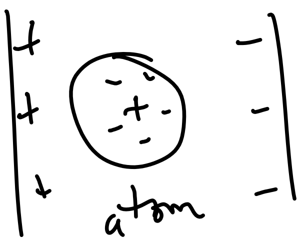
Electrical properties of matter



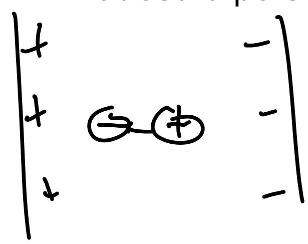


Questions that help us to generate a model:

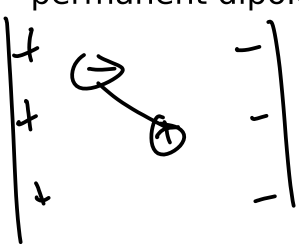
causal/creative: How can I simplify the problem to reveal the fundament physics?



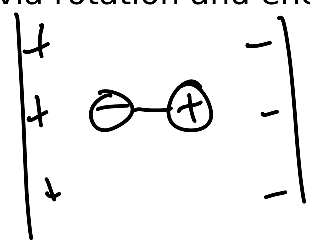
Induced dipole



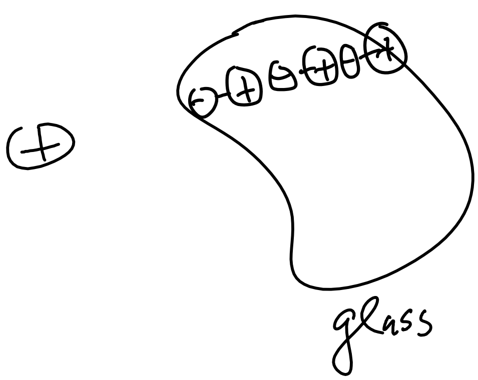
permanent dipole



alignment via rotation and energy dissapation



causal/creative: What happens to the atoms in the glass when a field is applied?

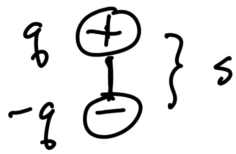


We will solve this problem by noting that the interior charges cancel leaving only surface charges. To find E get rid of the glass atoms and find the total E due to the surface charge and initial charge.

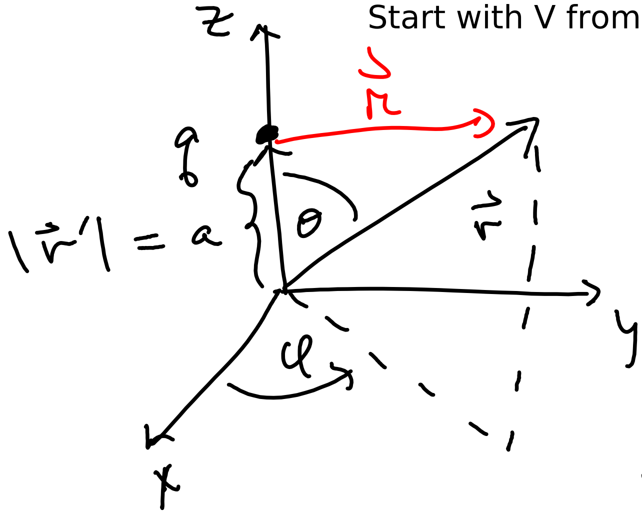
congruous: How can I perform a calculation or experiment to check this model?

Model: atoms are small and form dipoles when in an electric field. To find the net field in matter we need to sum the fields from all the dipoles along with the incident field. This is similar to what we did for a conductor.

Derivation of field from a dipole.



Start with V from an offset point charge q.



$$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$$

$$\vec{r}' = a\hat{z}$$

$$\vec{\mathscr{r}} = \vec{r} - \vec{r}' = x\hat{x} + y\hat{y} + (z-a)\hat{z}$$

$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{\mathscr{r}}$$

informational: How are we going to use this to get the potential from a dipole?

$$V = \frac{q}{4\pi\epsilon_0}\frac{1}{(r^2 + a^2 - 2ar\cos\theta)^{1/2}}$$

for $r > a$ expand in what?

$$\frac{1}{(r^2+a^2-2ar\cos\theta)^{1/2}} = \frac{1}{r(1+\delta)^{1/2}} = \frac{1}{r}\left[1-\frac{1}{2}\delta+\frac{3}{8}\delta^2+\ldots\right]$$

$$\delta = \frac{a^2}{r^2} - \frac{2ar\cos\theta}{r^2}$$

Arranging in powers of $\frac{a}{r} < 1$

$$\frac{1}{r}\frac{1}{(1+\delta)^{1/2}} = \frac{1}{r}\left[1+\frac{a}{r}\cos\theta+\left(\frac{a}{r}\right)^2\left\{\frac{3\cos^2\theta-1}{2}\right\}+\ldots\right]$$

$$= \frac{1}{r}\sum_{l=0}^{\infty}\left(\frac{a}{r}\right)^l P_l(\cos\theta) \qquad r>a$$

Legendre polynomials

congruous: How do I calculate the potential far away and does this make sense?

$$V = \frac{q}{4\pi\epsilon_0}\frac{1}{r}\left[1+\frac{a}{r}\cos\theta+\left(\frac{a}{r}\right)^2\left\{\frac{3\cos^2\theta-1}{2}\right\}+\ldots\right]$$

Axial multipole expansion. Let charge be distributed on the z axis.

Hmwk problem 1.) Derive an expansion for the potential given the following charge distribution:

$$\rho(r') = \delta(x')\,\delta(y')\,\lambda(z')$$

$$V = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r} \qquad dq = \rho\, d\tau$$

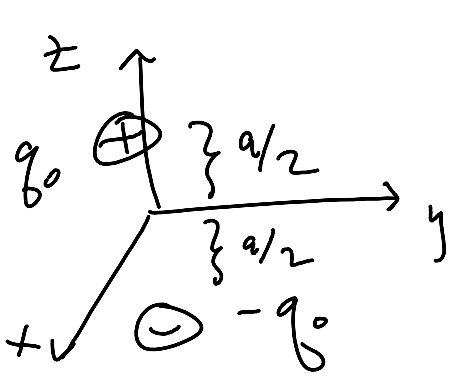
$$V = \frac{1}{4\pi\epsilon_0}\sum_{\ell=0}^{\infty} \frac{M_\ell P_\ell(\cos\theta)}{r^{\ell+1}} \;;\quad M_\ell = \int \lambda(z')(z')^\ell\, dz'$$

$$M_0 = \int \lambda(z')\, dz' = Q$$ monopole

$$M_1 = \int \lambda(z')\, z'\, dz'$$ dipole moment

$$M_0 = \int \lambda(z')\, dz' = Q \qquad V_0 = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$$

$$M_1 = \int \lambda(z')\, z'\, dz' \qquad V_1 = \frac{1}{4\pi\epsilon_0}\frac{M_1 \cos\theta}{r^2}$$



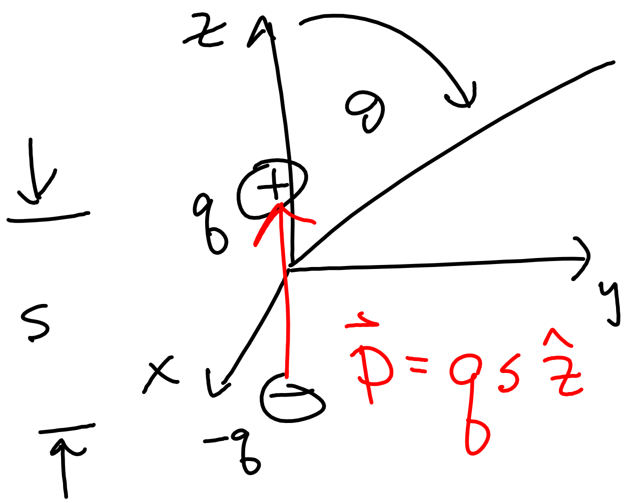
$$M_1 = \int \lambda(z')\, z'\, dz'$$

$$\lambda(z') = q_0\, \delta\!\left(z' - \frac{a}{2}\right) - q_0\, \delta\!\left(z' + \frac{a}{2}\right)$$

$$M_1 = q_0 \frac{a}{2} + q_0 \frac{a}{2} = q_0 a$$

$$V_1 = \frac{1}{4\pi\epsilon_0} \frac{qa\cos\theta}{r^2}$$

Hmwk problem 2.) Derive E starting with this expression for V and using the gradient in cartesian coordinates.

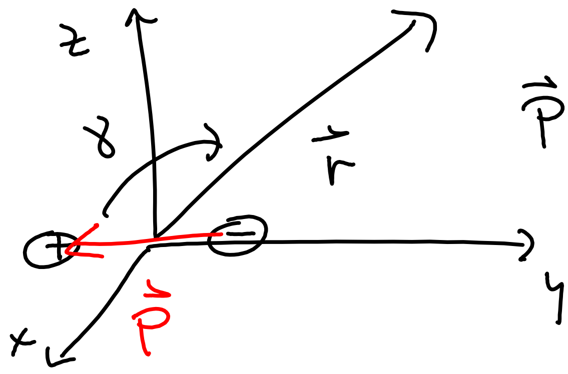


This dipole moment vector points from minus to plus and has magnitude qs.

$$V_1 = \frac{\vec{p}\cdot\hat{r}}{4\pi\epsilon_0 r^2} = \frac{|\vec{p}||\hat{r}|\cos\theta}{4\pi\epsilon_0 r^2} = \frac{qs\cos\theta}{4\pi\epsilon_0 r^2}$$

incongruous: How can this can't be the correct voltage because the voltage near the positive charge is proportional to 1/r?

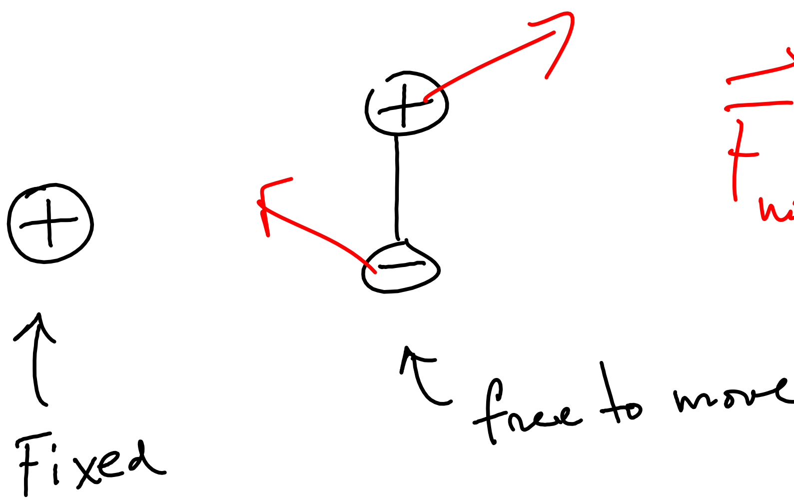
informational: Why use a vector representation?



$$\vec{p}\cdot\hat{r} = |\vec{p}|\underbrace{|\hat{r}|}_{=1}\cos\gamma$$

$$\gamma = 90 + \theta$$

Forces on dipoles.



$\vec{F}_{net}$ is up

$$\vec{F}_{net} = \vec{F}_+ + \vec{F}_- = q\left(\vec{E}_+ - \vec{E}_-\right) \rightarrow q\, d\vec{E}$$

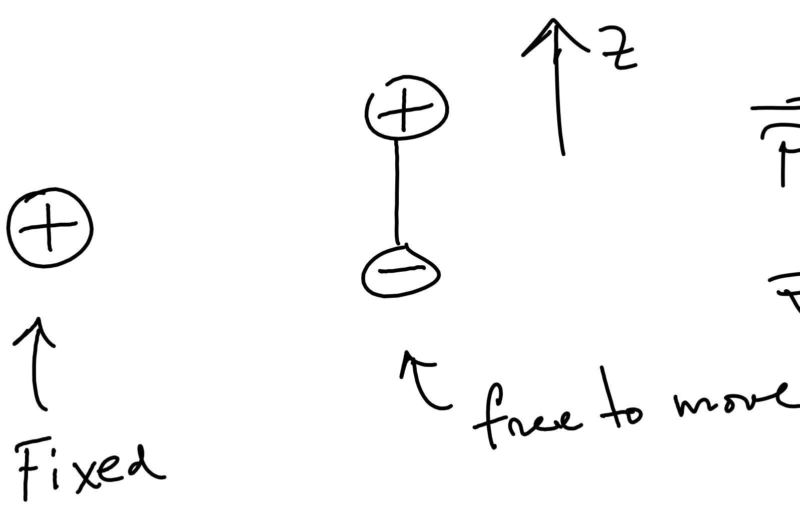
$$\vec{E}(x,y,z) = E_x(x,y,z)\,\hat{x} + E_y(x,y,z)\,\hat{y} + E(x,y,z)\,\hat{z}$$

$$dE_x = \frac{\partial E_x}{\partial x}dx + \frac{\partial E_x}{\partial y}dy + \frac{\partial E_x}{\partial z}dz$$

Hmwk problem 3: Expanding the differential for each component show that the force on the dipole can be written as

$$\vec{F} = \left(\vec{p}\cdot\vec{\nabla}\right)\vec{E}$$

congruous: How do I calculate the force on the dipole in the example above?

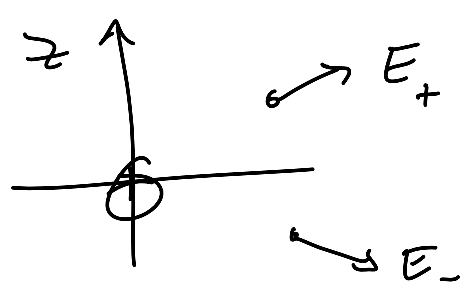


$$\vec{p} = p_0\,\hat{z}$$

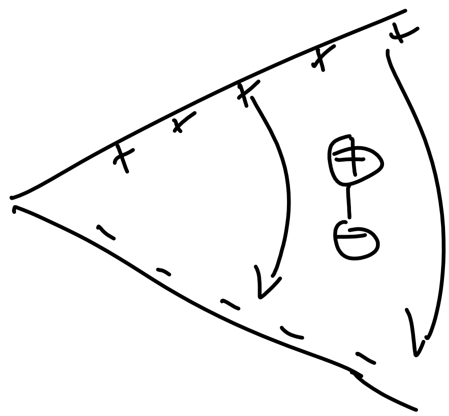
$$\vec{\nabla} = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$$

$$\vec{p}\cdot\vec{\nabla} = \frac{\partial}{\partial z}$$

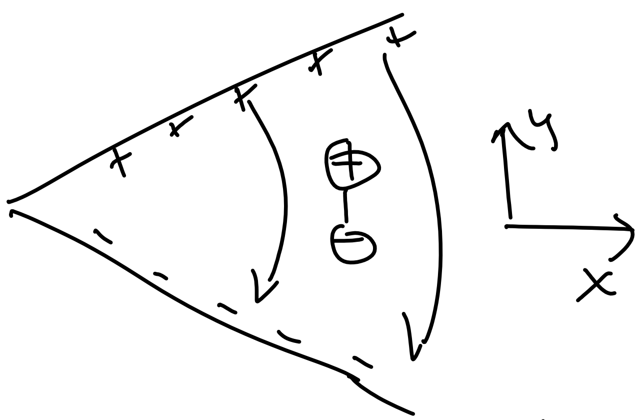
$$\left(\vec{p}\cdot\vec{\nabla}\right)\vec{E} \Rightarrow \frac{\Delta\vec{E}}{\Delta z}$$



Example



congruous: How do I calculate the force on the dipole?



$$\vec{p} = p_0 \hat{y}$$

$$p_0 \frac{\partial}{\partial y}\left(E_x \hat{x} + E_y \hat{y} + E_z \hat{z}\right)$$

$$= p_0 \frac{\partial}{\partial y}\left(E_x \hat{x} + E_y \hat{y}\right)\Big|_{y=0}$$

$$\frac{\partial E_x}{\partial y} = 0 \text{ ?}$$

Exam 3:

Grader average = 61 std dev =16

Student average= 70 std dev = 31

Difference between student and grader was as large as 50 pts.

You will get 10 points added to the graders score for participating as directed in this exercise. This is for the learning that occured while looking over you work.