$\boxed{P12}$ <u>Solutions</u>

(13.1) $n_{\binom{e}{h}} = \frac{1}{2}\left\{\sqrt{(\Delta n)^2 + 4n_i^2} \pm \Delta n\right\}$

$$|\Delta n| \ll n_i \Rightarrow \sqrt{(\Delta n)^2 + 4n_i^2} \approx 2n_i\left[1 + \frac{1}{8}\frac{(\Delta n)^2}{n_i^2}\right]$$

$\therefore n_e \approx n_i + \frac{\Delta n}{2} \qquad n_h \approx n_i - \frac{\Delta n}{2}$ (to <u>lowest order</u> in Δn)

(13.2)

(a) $\sigma = \sigma(\Delta n) \qquad \frac{d\sigma}{d\Delta n} = 0 = e\left(\tilde{\mu}_e \frac{dn_e}{d\Delta n} + \tilde{\mu}_h \frac{dn_h}{d\Delta n}\right)$

$\left(\sigma = \frac{ne^2\tau}{m}\right)$

$$\frac{dn_{e,h}}{d\Delta n} = \frac{d}{d\Delta n}\frac{1}{2}\left(\sqrt{(\Delta n)^2 + 4n_i^2} \pm \Delta n\right)$$

$$= \frac{1}{2}\left(\frac{\Delta n}{\sqrt{(\Delta n)^2 + 4n_i^2}} \pm 1\right)$$

$$0 = \tilde{\mu}_e\left(\frac{\Delta n}{\sqrt{(\Delta n)^2 + 4n_i^2}} + 1\right) + \tilde{\mu}_h\left(\frac{\Delta n}{\sqrt{(\Delta n)^2 + 4n_i^2}} - 1\right)$$

$$\left(\sqrt{} + \Delta n\right)\tilde{\mu}_e = \tilde{\mu}_h\left(\sqrt{} - \Delta n\right) \Rightarrow n_e\tilde{\mu}_e = n_h\tilde{\mu}_h$$

$$(\tilde{\mu}_e - \tilde{\mu}_h)\sqrt{} = -(\tilde{\mu}_h + \tilde{\mu}_e)\Delta n$$

(note: with $\tilde{\mu}_e > \tilde{\mu}_h$, need $\Delta n < 0$; $\tilde{\mu}_h > \tilde{\mu}_e$, need $\Delta n > 0$)

$$(\Delta n)^2 + 4n_i^2 = \left(\frac{\tilde{\mu}_e + \tilde{\mu}_h}{\tilde{\mu}_e - \tilde{\mu}_h}\right)^2 \Delta n^2 \Rightarrow |\Delta n| = \frac{2n_i}{\sqrt{\left(\frac{\tilde{\mu}_e + \tilde{\mu}_h}{\tilde{\mu}_e - \tilde{\mu}_h}\right)^2 - 1}}$$

$$\boxed{|\Delta n| = \frac{n_i}{\sqrt{\tilde{\mu}_e\tilde{\mu}_h}}|\tilde{\mu}_e - \tilde{\mu}_h|} \text{ or } \boxed{\Delta n = \frac{n_i(\tilde{\mu}_h - \tilde{\mu}_e)}{\sqrt{\tilde{\mu}_e\tilde{\mu}_h}}}$$

$$\boxed{\sigma_{min} = 2en_e\tilde{\mu}_e} = 2en_i\sqrt{\tilde{\mu}_e\tilde{\mu}_h}$$

(b) For $\Delta n = 0$ (intrinsic case) $n_e = n_h = n_i$ and $\sigma_i = 2en_i(\tilde{\mu}_e + \tilde{\mu}_h)$

$$\therefore \frac{\sigma_{min}}{\sigma_i} = \frac{\sqrt{\tilde{\mu}_e\tilde{\mu}_h}}{\tilde{\mu}_e + \tilde{\mu}_h}$$

(c)

$$\sigma_{min} = 2e\,\tilde{\mu}_e n_i \sqrt{\frac{\tilde{\mu}_h}{\tilde{\mu}_e}} = 2e\sqrt{\tilde{\mu}_e \tilde{\mu}_h}\; n_i \quad \leftarrow \text{units: } \frac{\text{Coul}}{\text{V-sec-cm}} = \frac{1}{\text{ohm-cm}}$$

$$n_i = \sqrt{n_c n_v}\; e^{-\mathcal{E}_g/2\tau} \qquad e = 1.9 \times 10^{-19} \text{ Coul}$$

<u>Si at T=300 K</u> $\quad n_c = 2.7 \times 10^{19}\ \text{cm}^{-3} \quad n_v = 1.1 \times 10^{19}\ \text{cm}^{-3}$

$$\mathcal{E}_g/2 = 0.57\ \text{eV (at 300 K)}$$

$$\tau = \frac{300}{1.16} \cdot 10^{-4}\ \text{eV} \cong 2.6 \times 10^{-2}\ \text{eV} = 0.026\ \text{eV}$$

$$n_i = \sqrt{n_c n_v}\; e^{-\mathcal{E}_g/2\tau} = \sqrt{(2.7)(1.1)} \cdot 10^{19} \exp(-0.57/0.026) = 1.72 \cdot 10^{19} \cdot 10^{-21.9/2.30}$$

$$= 1.72 \cdot 10^{19-9.52} = 3.32^{-1} \times 1.72 \times 10^{10}$$

$$\boxed{\sigma_{min}(Si) = 1.34 \times 10^{-6}\ (\text{ohm-cm})^{-1}}$$

InSb at T=300 K $\quad n_c = 4.6 \times 10^{16}\ \text{cm}^{-3} \quad n_v = 6.2 \times 10^{18}\ \text{cm}^{-3}$

$$\mathcal{E}_g/2 = 0.09\ \text{eV}$$

$$\tau = 0.026\ \text{eV}$$

$$\boxed{\sigma_{min}(\text{InSb}) \simeq 40.75\ (\text{ohm-cm})^{-1}}$$

(13.3) $\left(\text{ohm} = \frac{\text{V-sec}}{\text{coul}}\right)$

$$\sigma = \frac{1}{20} = e\,(3900\, n_e + 1900\, n_h)$$

$$= e \cdot 10^3 (3.9 n_e + 1.9 n_h) = 10^3 e\left[1.95\left(\sqrt{(\Delta n)^2 + 4n_i^2} + \Delta n\right) + 0.95\left(\sqrt{(\Delta n)^2 + 4n_i^2} - \Delta n\right)\right]$$

Since τ is <u>not specified</u>, must assume $|\Delta n| \gg n_i$, so

$$\sigma = \frac{1}{20} \approx \begin{cases} 10^3 e\left(3.9|\Delta n| + 1.9\,\dfrac{2n_i^2}{|\Delta n|}\right) \approx 3.9\, e|\Delta n| \cdot 10^3 & \text{n-type} \\ 10^3 e\,(1.9|\Delta n|) & \text{p-type} \end{cases}$$

$$|\Delta n| = \begin{cases} \dfrac{10^{16}}{(1.9)(3.9)(20)}\ \text{cm}^{-3} = 6.75 \cdot 10^{13}\ \text{cm}^{-3} & \text{n-type} \\ \dfrac{10^{16}}{(1.9)(1.9)\,20}\ \text{cm}^{-3} = 1.39 \cdot 10^{14}\ \text{cm}^{-3} & \text{p-type} \end{cases}$$

(13.7) $n_1, n_2 \gg n_i \Rightarrow n_a^- \gg n_i \Rightarrow |\Delta n| \gg n_i$

$$\Rightarrow n_h = n_a^-$$

$\therefore$

$$n_h(x) = n_a^-(x) = n_v e^{(\varepsilon_v - \mu - e\phi(x))/\tau}$$

$$n_a^-(x_1) = n_1 = n_v e^{(\varepsilon_v - \mu - e\phi(x_1))/\tau}$$

$$n_a^-(x_2) = n_2 = n_v e^{(\varepsilon_v - \mu - e\phi(x_2))/\tau}$$

$$\frac{n_1}{n_2} = e^{e(\phi(x_2) - \phi(x_1))/\tau}$$

$$\frac{\phi(x_2) - \phi(x_1)}{x_2 - x_1} = -E = \frac{\tau/e \ln\left(\frac{n_1}{n_2}\right)}{x_2 - x_1} \simeq +1.8 \times 10^4 \ \frac{V}{cm}$$

(13.8) D_e is defined by

$$e D_e \vec{\nabla} n_e = \tilde{\mu}_e n_e \tau \vec{\nabla} \eta \qquad \left(\eta \equiv \frac{\mu - \varepsilon_c}{\tau}\right)$$

$$\eta \approx \ln\left(\frac{n_e}{n_c}\right) + \frac{1}{\sqrt{8}} \frac{n_e}{n_c} - \left(\frac{3}{16} - \frac{\sqrt{3}}{9}\right)\left(\frac{n_e}{n_c}\right)^2$$

$$\therefore \ e D_c \vec{\nabla} n_e = \tilde{\mu}_e n_e \tau \left[\frac{\vec{\nabla} n_e}{n_e} + \frac{1}{\sqrt{8}} \frac{1}{n_c} \vec{\nabla} n_e - 2\left(\frac{3}{16} - \frac{\sqrt{3}}{9}\right) \frac{n_e}{n_c^2} \vec{\nabla} n_e \right]$$

$$\Rightarrow e D_e = \tilde{\mu}_e \tau \left(1 + \frac{1}{\sqrt{8}} \frac{n_e}{n_c} - 2\left(\frac{3}{16} - \frac{\sqrt{3}}{9}\right)\left(\frac{n_e}{n_c}\right)^2 \right)$$

(13.10)

$$R = \frac{n_{eo}\delta n + n_{ho}\delta n + \delta n^2}{(n_e^* + n_{eo} + \delta n)t_h + (n_h^* + n_{ho} + \delta n)t_e} \equiv \frac{\delta n}{t} \qquad (n_{eo} n_{ho} = n_i^2)$$

$$\frac{1}{t} = \frac{n_{eo} + n_{ho} + \delta n}{(n_e^* + n_{eo} + \delta n)t_h + (n_h^* + n_{ho} + \delta n)t_e}$$

$\delta n \ll n_{eo}, n_{ho}$

$$\frac{1}{t} \approx \frac{n_{eo} + n_{ho}}{(n_e^* + n_{eo})t_h + (n_h^* + n_{ho})t_e} = \frac{1}{\left(\frac{n_e^* + n_{eo}}{n_{eo} + n_{ho}}\right)t_h + \left(\frac{n_h^* + n_{ho}}{n_{eo} + n_{ho}}\right)t_e}$$

$\delta n \gg n_{eo}, n_{ho}$

$$\frac{1}{t} \approx \frac{1}{t_h + t_e}$$

t is independent of δn in <u>both</u> of these limits.