

1.1.4 Determine the conditions under which each of the following conditional sentences will be a true statement.

1. If $a + 2 = 5$, then $8 < 5$
2. If $5 < 8$, then $a + 2 = 5$

1.1.6 Following is a statement of a theorem which can be proven using calculus or precalculus mathematics. For this theorem, a , b , and c are real numbers.

Theorem If f is a quadratic function of the form $f(x) = ax^2 + bx + c$ and $a < 0$,
then the function f has a maximum value when $x = \frac{-b}{2a}$.

Using only this theorem, what can be concluded about the functions given by the following formulas?

1. $g(x) = -8x^2 + 5x - 2$
2. $f(x) = -4x^2 - 2x + 7$
3. $F(x) = -x^4 + x^3 + 9$

1.1.7 Following is a statement of a theorem which can be proven using the quadratic formula. For this theorem, a , b , and c are real numbers.

Theorem If f is a quadratic function of the form $f(x) = ax^2 + bx + c$ and $ac < 0$,
then the function f has two x -intercepts. $x = \frac{-b}{2a}$.

Using only this theorem, what can be concluded about the functions given by the following formulas?

1. $g(x) = -8x^2 + 5x - 2$
2. $f(x) = -4x^2 - 2x + 7$
3. $F(x) = -x^4 + x^3 + 9$

1.1.8 Following is a statement of a theorem about certain cubic equations. For this theorem, b represents a real number.

Theorem A If f is a cubic function of the form $f(x) = x^3 - x + b$ and $b > 1$, then the function f has exactly one x -intercept.

Following is another theorem about x -intercepts of functions:

Theorem B If f and g are functions with $g(x) = k \cdot f(x)$ where k is a nonzero real number, then f and g have exactly the same x -intercepts.

Using only these two theorems and some simple algebraic manipulations, what can be concluded about the functions given by the following formulas?

1. $f(x) = x^3 - x + 7$
2. $r(x) = x^4 - x + 11$
3. $F(x) = 2x^3 - 2x + 7$

1.2.4 Write a complete proof for the following statement:

- a. If m is an even integer, then $5m + 7$ is an odd integer.

1.2.5 Write a complete proof for the following statement:

- b. If m is an odd integer, then $3m^2 + 7m + 12$ is an even integer.