P1 Solutions

1. (a) $\sigma(N,U) = \log(C\,U^{3N/2})$

$$\frac{1}{\tau} = \left(\frac{\partial\sigma}{\partial U}\right)_N = \frac{3N}{2}\frac{1}{U}$$

(b) $$\left(\frac{\partial^2\sigma}{\partial U^2}\right)_N = \left(\frac{\partial\left(\frac{3N}{2}\frac{1}{U}\right)}{\partial U}\right)_N = -\frac{3N}{2}\frac{1}{U^2} < 0$$

2. $U = -2s\,mB$

$$\sigma(s) \approx \log g(N,0) - \frac{2s^2}{N} = \log g(N,0) - \frac{U^2}{2(mB)^2 N}$$

$$\frac{1}{\tau} = \left(\frac{\partial\sigma}{\partial U}\right)_N = \frac{-\langle U\rangle}{(mB)^2 N} \qquad \langle U\rangle = -2\langle s\rangle mB = \frac{(mB)^2 N}{\tau}$$

gives $U = \langle U\rangle$ as a function of τ at equilib. $\therefore$ $\langle s\rangle = -\frac{mBN}{2\tau}$

$$\Rightarrow \frac{2\langle s\rangle}{N} = -\frac{mB}{\tau}$$

3. Neglecting zero point energy: $U = n\hbar\omega$

(a) Multiplicity function (cf pp. 24–25 of text):

$$g(N,n) = \frac{(N+n-1)!}{n!(N-1)!}$$

$$\sigma = \log g(N,n) = (N+n-1)\log(N+n-1) - \cancel{(N+n-1)} - n\log n + \cancel{n}$$
$$-(N-1)\log(N-1) + \cancel{N-1}$$
$$= (N+n-1)\log(N+n-1) - n\log n - (N-1)\log(N-1)$$

(b) $$\sigma(U,N) = \left(N+\frac{U}{\hbar\omega}-1\right)\log\left(N+\frac{U}{\hbar\omega}-1\right) - \frac{U}{\hbar\omega}\log\left(\frac{U}{\hbar\omega}\right) - (N-1)\log(N-1)$$

$$\frac{1}{\tau} = \left(\frac{\partial\sigma}{\partial U}\right)_N = \cancel{\frac{1}{\hbar\omega}} + \frac{1}{\hbar\omega}\log\left(N+\frac{\langle U\rangle}{\hbar\omega}-1\right) - \cancel{\frac{1}{\hbar\omega}} - \frac{1}{\hbar\omega}\log\left(\frac{\langle U\rangle}{\hbar\omega}\right)$$

$$\frac{\hbar\omega}{\tau} = \log\left(\frac{(N-1)\hbar\omega + \langle U\rangle}{\langle U\rangle}\right) \Rightarrow e^{\hbar\omega/\tau}\langle U\rangle = (N-1)\hbar\omega + \langle U\rangle$$

$$\langle U\rangle = \frac{(N-1)\hbar\omega}{e^{\hbar\omega/\tau}-1} \approx \frac{N\hbar\omega}{e^{\hbar\omega/\tau}-1} \qquad (N >\!>\!> 1)$$