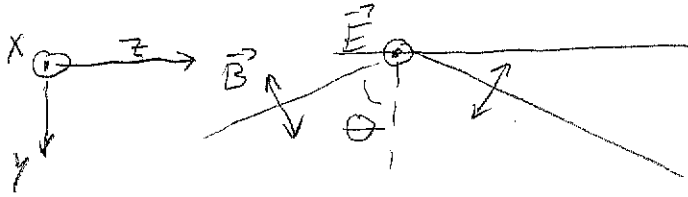


Parallel plane waveguide - conducting  
 works with  $\vec{E}$  tangential,  $\vec{B}^\perp \rightarrow 0$  at surface.  
 - perfect conductor has free charges + currents.  
 - this choice allows us to avoid working with them

Free space waves are TEM:  $\vec{E} \perp \vec{B} \perp \vec{k}$ , all compon transverse



TE:  $\vec{B}$  has y, z compon.  
 $\vec{E}$  is only x

TM:  $\vec{B}$  is transverse  
 solve for each case separately.

TE example:

construct solution from plane waves.

$$\vec{E}_+ = +\hat{x} E_0 e^{i(k_y y + k_z z)} \quad \text{with } k_y = k_0 \cos \theta$$

$$k_z = k_0 \sin \theta$$

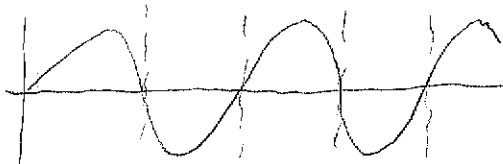
$$\vec{E}_- = -\hat{x} E_0 e^{i(+k_y y + k_z z)}$$

$$\vec{E}_+ + \vec{E}_- = 0 \quad \text{at } y=0$$

$$= \hat{x} E_0 e^{ik_z z} (e^{-ik_y y} - e^{+k_y y}) \quad 0 < y < b$$

$$= -2i E_0 \sin(k_y y) e^{ik_z z}$$

traveling wave in z  
 standing wave in y



could place second surface at any node w/o change to wave.

$$\sin k_y b = 0 \quad k_y b = k_0 b \cos \theta = n\pi \quad n \text{ integer } \geq 1$$

$$\text{given } \lambda \text{ or } \omega: \text{ allowed } \theta\text{'s} \quad \cos \theta_n = \frac{n\pi}{k_0 b}$$

$$\text{given } \theta: \text{ allowed freq} \quad \omega_n = \frac{n\pi c}{b \cos \theta}$$

the wave now has a fixed profile in  $y$  direction  
travels in  $z$ -direction:

$$\vec{E} = \sum E(y) e^{i(k_y z - \omega t)}$$

with guide wavenumber  $k_g = k_z = \sqrt{k_0^2 - k_y^2}$

discrete values of  $k_y = n\pi/b$

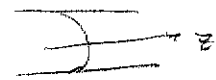
→ discrete values of  $k_g$ :

$$k_g = \sqrt{k_0^2 - (n\pi/b)^2} \quad \text{allowed values.}$$

cutoff frequency: express  $k_0 = \omega/c$

$$k_g = \sqrt{\omega^2/c^2 - (n\pi/b)^2}$$

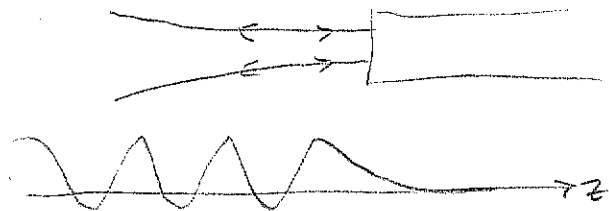
At  $n=1$ , transverse mode is:



at  $\omega = \omega_c = \pi c/b$ , cutoff frequency for  $n=1$

$$\rightarrow k_g = 0$$

for  $\omega < \omega_c$   $k_g \rightarrow$  pure imag., input wave reflects



can express cutoff in terms of  $\nu_c = \frac{\omega_c}{2\pi} = \frac{nc}{2b}$   
each mode index has a cutoff.

if input  $\omega$  satisfies  $\frac{\pi c}{b} < \omega < \frac{2\pi c}{b}$   
guide allows only one mode.  
"single-mode"

Propagation of waves in waveguide,

for simplicity, consider one transverse dimension

$$\vec{E}(y, z, t) = \hat{x} E_0 \sin\left(\frac{n\pi y}{b}\right) e^{i(k_z z - \omega t)}$$

phase velocity  $v_{ph} = \frac{\omega}{k_z}$

remember  $k_z = \sqrt{k_x^2 - k_y^2} = \sqrt{\omega^2/c^2 - \left(\frac{n\pi}{b}\right)^2}$

$$v_{ph} = \frac{\omega}{\sqrt{\omega^2 - \left(\frac{n\pi c}{b}\right)^2}} \cdot c > c$$

group velocity

$$v_g = \frac{d\omega}{dk_z} = \left(\frac{dk_z}{d\omega}\right)^{-1}$$

$$= \left[ \frac{1}{2} \cdot \frac{2\omega}{c^2} \left( \frac{\omega^2}{c^2} - \left(\frac{n\pi}{b}\right)^2 \right)^{-1/2} \right]^{-1}$$

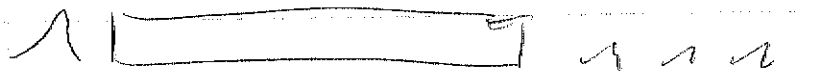
$$= \frac{c^2}{\omega} \cdot \frac{\omega}{c} \sqrt{1 - \left(\frac{n\pi c}{\omega b}\right)^2} \quad \text{less than } c$$

$$= c \sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}$$

modal dispersion:

dependence of  $v_{ph}$ ,  $v_g$  on mode index

multimode propagation:



each mode travels w/ diff't speed.

# Metal Waveguides

including vector character and boundary cond.  
 vectors: anticipate orientation of fields rel. to walls.  
 work solution

TEM  $\vec{E} \perp \vec{B} \perp \hat{z}$        $E_z = B_z = 0$

example: free space plane wave.

TE  $\vec{E}$  is transverse only       $\vec{E}_z = 0$



$B_z \neq 0$

TM  $B_z = 0$

"transverse" means field is  $\perp \hat{z}$

Remember, once modes are known (eigenfunctions) an arbitrary solution can be composed

$$\vec{E} = \sum_m a_m E_{TE_m} + \sum_n b_n E_{TM_n}$$

modes are a complete set that can describe any function compatible with boundary conditions.

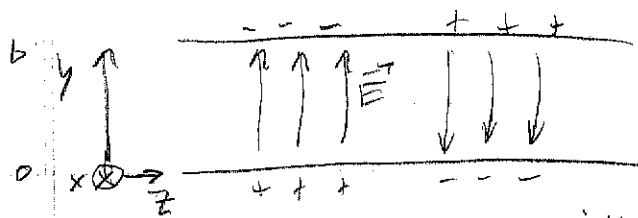
Strip lines: plane parallel conductors

assume perfect conductor  $\rightarrow$  surface charge, constant.

TEM is allowed:  $\frac{E}{\perp \vec{B}} \rightarrow \hat{z}$

same velocity as in vacuum (or air)

no modes.



For TEM  $\vec{E} = \hat{y} E_0 e^{i(kz - \omega t)}$  along  $\hat{z}$   $E \parallel$  to metal is zero.

Since  $\nabla \cdot \vec{E} = 0$  inside w/g,

$$\rightarrow \frac{\partial E}{\partial y} = 0 \quad \vec{E} \text{ is constant}$$

finite value of  $E_z$  at metal  $\rightarrow$  surf. charge.

- plates are not connected electrically.

get  $\vec{B}$  from  $\vec{E}$ :

$$\nabla \times \vec{E} = i\omega \vec{B}$$

$E$  varies only w/  $z$ .

$$-ikE_0 \hat{x} = i\omega \vec{B}$$

$$\begin{matrix} x & y & z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ & & E_y \end{matrix}$$

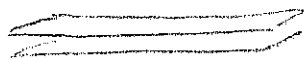
$$\vec{B} = -\hat{x} E_0$$

$$\omega = kc$$

from wave eqn.

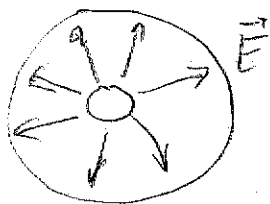
Wave behaves similar to free wave, but  $\vec{E}$  only along  $y$ ,  
 $\vec{E}$  is confined btw. plates.

in practice: striplines



- no cutoff i. prop. long  $\lambda$  in small guide.

coax. cable:



wave velocity determined by dielectric between conductors.

TE modes

$\vec{E}$  is  $\perp$  propagation direction: either  $\hat{x}, \hat{y}$

$$\nabla \cdot \vec{E} = 0 \quad E_z = 0 \quad \text{and} \quad \partial_x = 0$$

$$\rightarrow \partial_y E_y = 0 \quad \rightarrow E_y = \text{const. on } 0$$

$$\therefore \vec{E} = \hat{x} E_x(y) e^{i(k_z z - \omega t)} \quad \begin{matrix} \rightarrow \text{TEM} \\ \rightarrow \text{TE} \end{matrix}$$

solve for  $\vec{E} \rightarrow \vec{B}$  thru  $\nabla \times \vec{E} = -\frac{1}{c} \partial_t \vec{B} = \frac{i\omega}{c} \vec{B}$

$$\begin{vmatrix} x & y & z \\ \partial_x & \partial_y & \partial_z \\ E_x(y) & 0 & 0 \end{vmatrix}$$

$$\hat{y} \partial_z E_x - \hat{z} \partial_y E_x = \frac{i\omega}{c} \vec{B}$$

two components of  $\vec{B}$

since  $E_x(0) = E_x(b) = 0$

$$\rightarrow E_x(y) = E_0 \sin\left(\frac{n\pi y}{b}\right) \quad \rightarrow k_{yn} = n\pi/b$$

$$\vec{B} = -\frac{ic}{\omega} \left( i k_z E_0 \sin(k_{yn} y) \hat{y} - k_{yn} E_0 \cos(k_{yn} y) \hat{z} \right)$$

notes: as mode index  $n \uparrow$   $B_z$  is larger

factor of  $i$  on  $B_z \rightarrow$  phase shift

$$i e^{i(k_z z - \omega t)} \rightarrow \sin(k_z z - \omega t)$$

Power Flow:

$$\vec{S} = \frac{c}{4\pi} \vec{E} \times \vec{B} \quad (n=1) = \frac{c}{4\pi} \begin{vmatrix} x & y & z \\ E_x & 0 & 0 \\ 0 & B_y & B_z \end{vmatrix}$$

$$= \frac{c}{4\pi} \left( \hat{y} E_x B_z + \hat{z} E_x B_y \right)$$

instantaneously  $\vec{S}$  not always along

$\hat{z}$  - avg - only  $\hat{z}$  component  $\sim \cos^2(k_z z - \omega t)$

$\hat{y}$  comp  $\sim \cos \rightarrow \sin$