

Jan 16

Sec 1-4  $\frac{1}{\epsilon_0}$  2-4  $\frac{1}{\epsilon_0}$  conservation of charge

**(1) Gauss's Law**

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{\text{enclosed}}}{\epsilon_0}$$

**Incongruous:**

Doesn't it work only for symmetric shapes?

**Congruous:**

How is it applied to an egg shaped surface?

How does it relate to the divergence theorem?

**Modifying:**

Moving charge does Gauss's law still hold?



How does medium effect gauss's law?

**Generalizing/Analogy:**

Is there a Gauss's law for gravity?

**Causal/Creative:**

Why does it work?

**Informational:**

Why does it need a closed surface?

Any size restrictions on it application?

## (2) Conservation of charge

Expressed as a PDE using flux (chapter 9 in Shadowitz for conductors)

Flow of charge thru a surface

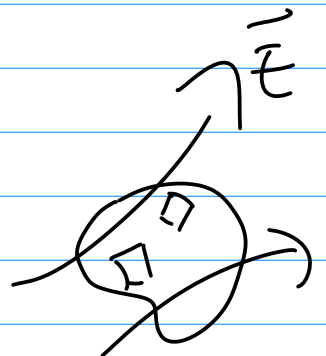
$$= \underbrace{\int \vec{J} \cdot d\vec{a}}_{\text{flux: } \frac{\text{Coulombs}}{\text{s}}} = \vec{J} \equiv \rho \vec{v} \quad \begin{array}{l} \frac{\text{Coulombs}}{\text{m}^3} \frac{\text{m}}{\text{s}} \\ \frac{\text{C}}{\text{s}} \frac{1}{\text{m}^2} \end{array}$$

$$\oint \vec{J} \cdot d\vec{a} \quad \text{NET flow of charge IN/OUT of closed surface}$$

↑ closed surface

There is no source or sink of charge so the net charge coming out of surface must be due to a change in charge within the surface

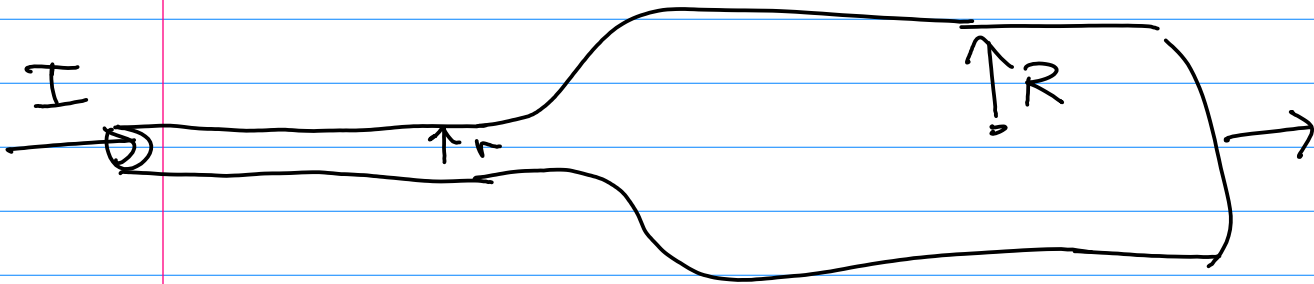
$$\oint \vec{J} \cdot d\vec{a} = - \frac{dQ_{\text{enclosed}}}{dt}$$
$$\oint \vec{E} \cdot d\vec{a} = Q_{\text{enc}} / \epsilon_0$$



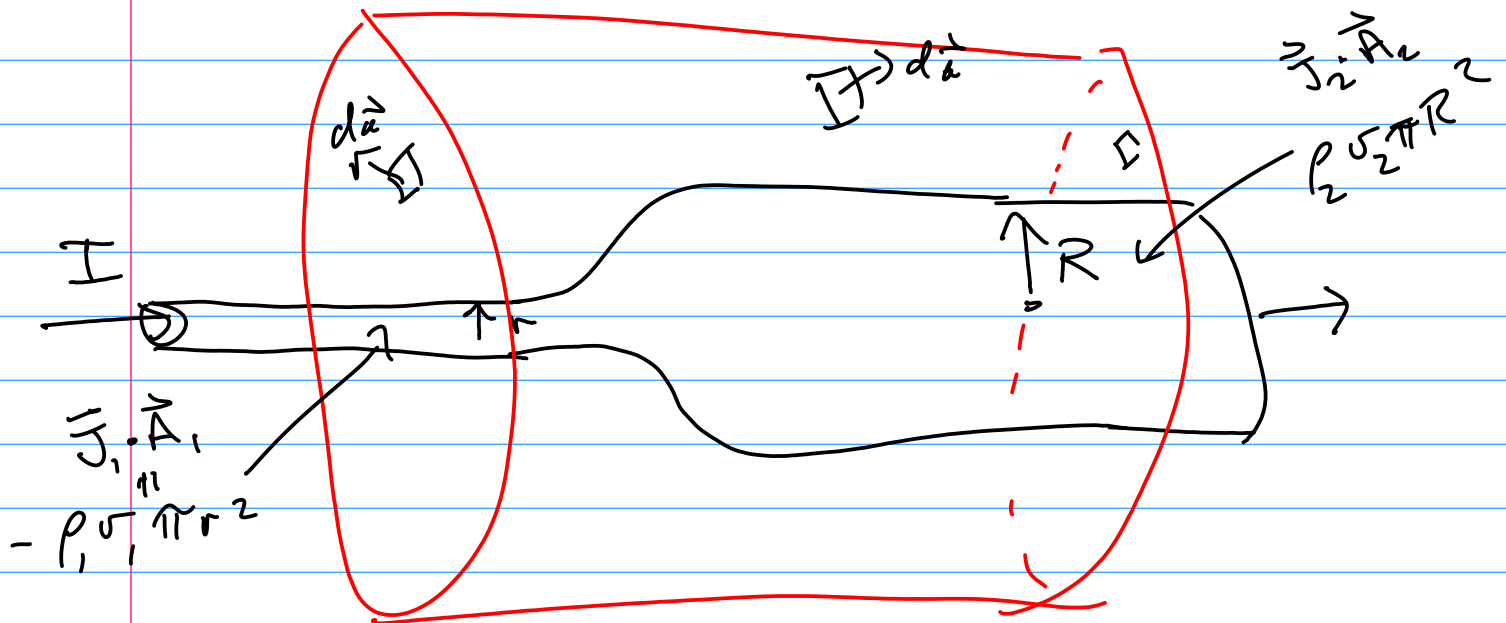
Is there a piling up of electric field lines in a closed surface?

In a conductor  $\rho \approx \text{constant}$

$$\int \vec{J} \cdot d\vec{a} = \int \rho \vec{v} \cdot d\vec{a} \quad \frac{C}{s} \text{ or Amps}$$



Informational: If the conductor is neutral what does the charge density mean?



$$\oint \vec{J} \cdot d\vec{a} = \frac{dQ_{\text{enclosed}}}{dt} = 0$$

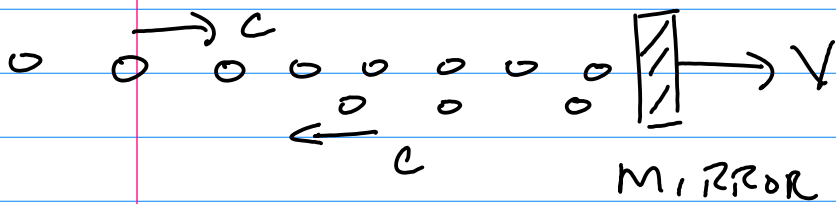
$$-\vec{J}_1 \cdot \vec{A}_1 + \vec{J}_2 \cdot \vec{A}_2 = 0 : \rho_1 v_1 \pi r^2 = \rho_2 v_2 \pi R^2$$

Congruous: How do I calculate the consequences of conservation of charge?

Informational: Does charge build up within the closed surface?

**Example: conservation of photons in a laser beam reflecting from a moving mirror**

Photons reflecting as a stream of particles 1-D



$$\vec{J}_{in} = \rho_{in} \vec{v}_{in} \rightarrow \lambda_{in} c$$

$$J_{reflected} = \rho_{ref} \vec{v}_{ref} \rightarrow \lambda_{ref} c$$

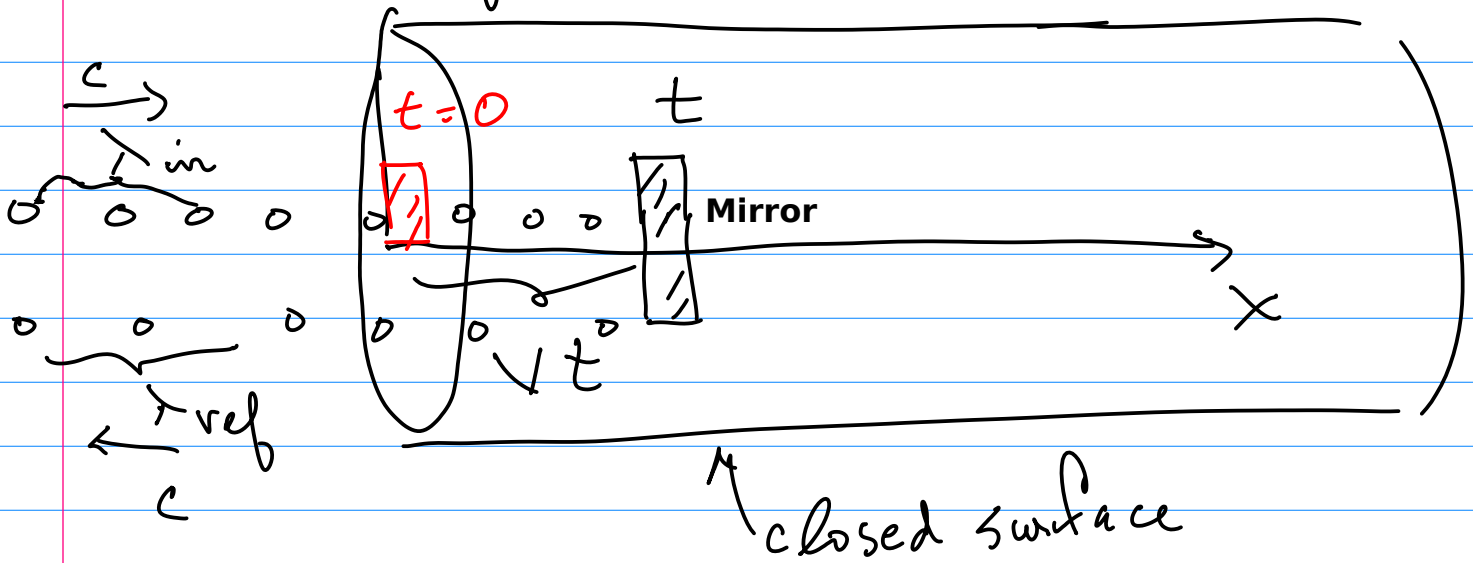
Model

$$\vec{J}_{in} = \lambda_{in} c \hat{x}$$

$$\vec{J}_{ref} = -\lambda_{ref} c \hat{x}$$

Reality

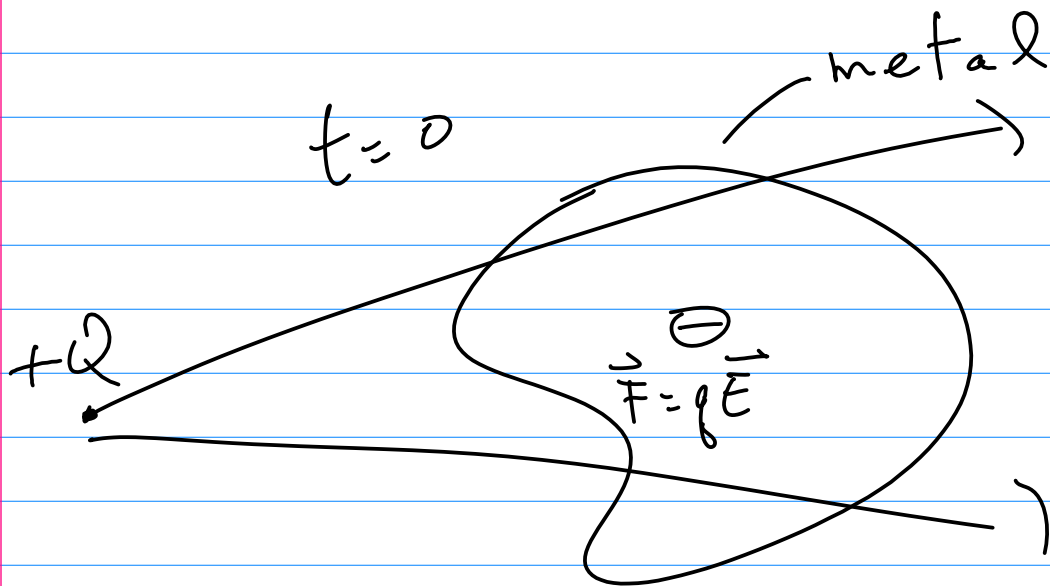
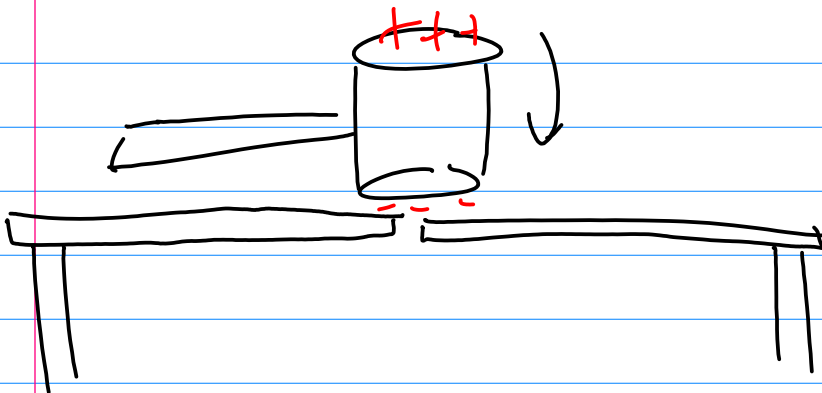
$$10^{14} \frac{\text{photons}}{\text{sec}} \pm \sqrt{10^{14}}$$



Conservation eqn ?

### (3) Conductors

Some electrons are free to move



**Congruous: How are the charges distributed using Coulomb's law?**

