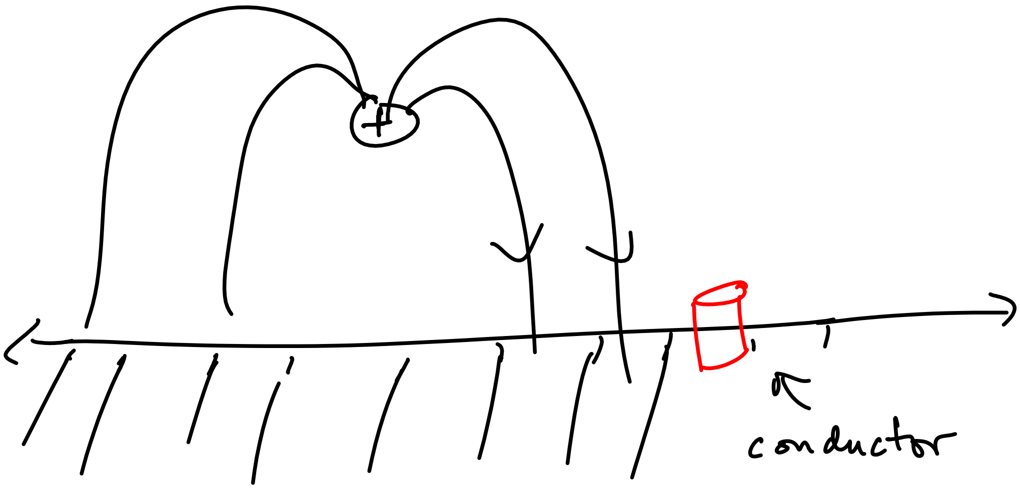
Lecture 39

Last week of classes: PDE boundary conditions applied to E and B, magnetic circuits, design problem, and possibly separation of variables. Homework due Wednesday April 30.

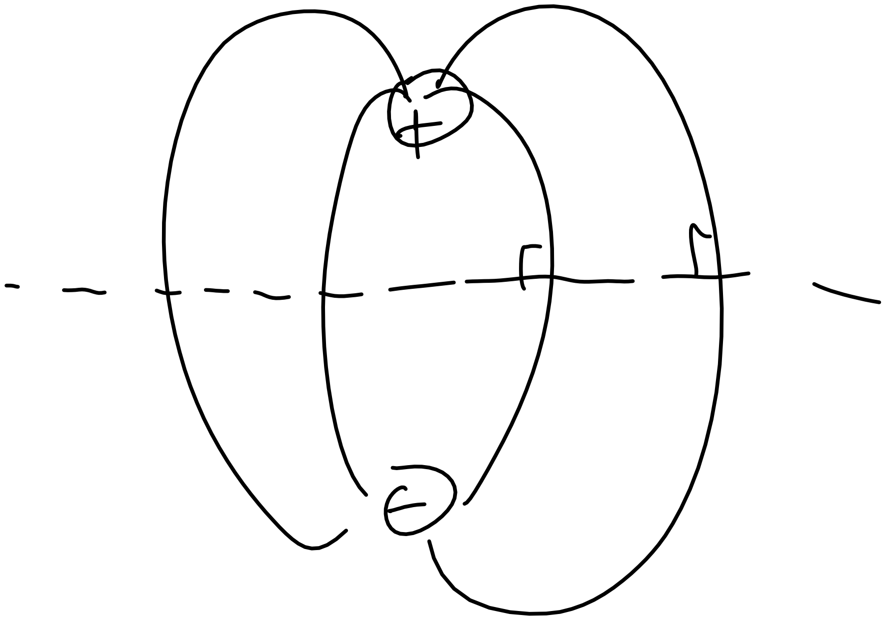
PDE + boundary conditions must yield a unique solution since nature does not sometimes do one thing and then another (no multiple solutions). (incongruous: what about QM?)

Example:



What's the boundary condition?

$$EA = \frac{\sigma A}{\epsilon_0}$$



$$V_{tot} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r} - \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$$

Homework problem 1.) Sketch the charge density as a function of position for a charge Q a distance D above an infinite conductor.

Homework problem 2.) A thermonuclear weapon was detonated underground 30 years ago. The radioactive lifetime of the remants is thousands of years so the heat energy generated can be accurately modeled as being steady. It can be shown that the PDE for the steady state temperature distribution in the Earth is

$$k\nabla^2 T = g$$

where k is the thermal conductivity and g is the heat source energy per volume.

$$\nabla^2 V = -\rho/\epsilon_0$$

This is Poisson's equation. We can model the surface of the Earth above the detonation as not allowing any thermal energy to flow out of it. The parameters in electrostatics V and E are the analogs of T and g where the flow of heat energy per time per area h is

$$\vec{h} = -k\vec{\nabla}T$$

$$\vec{E} = -\vec{\nabla}V$$

The boundary condition for the heat problem is that no flow of heat energy per second from the surface of the Earth. This is analogous to no component of the electric field perpendicular to a similar surface.

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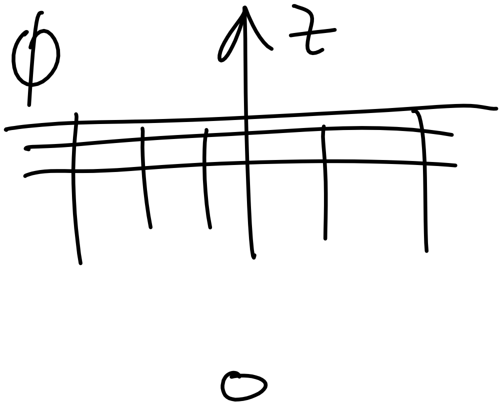
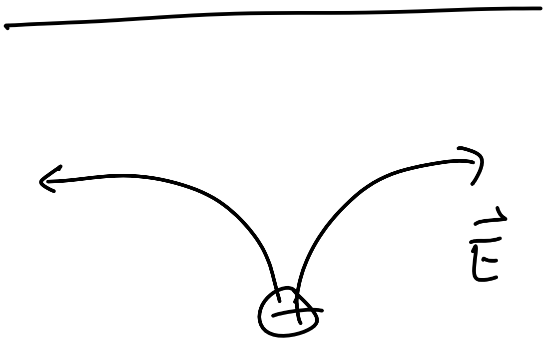
$$\frac{\partial V}{\partial z} = 0$$

$$E_\perp = 0 \qquad \vec{E} = -\vec{\nabla}V$$



heat source

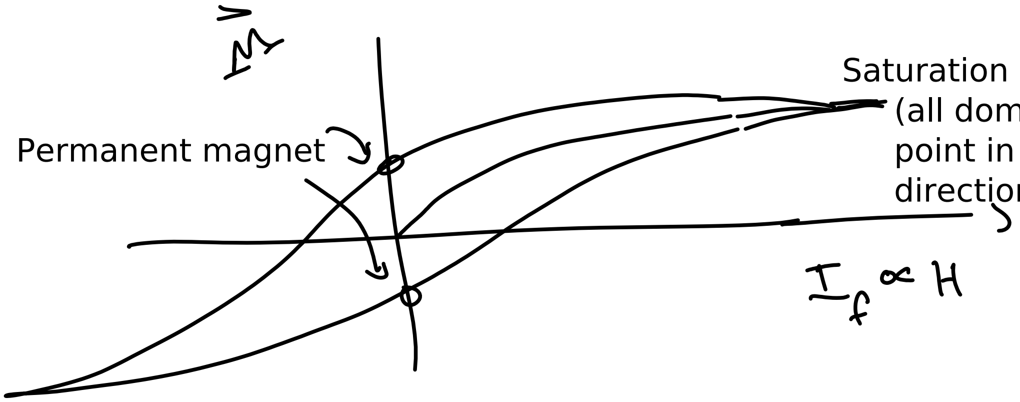


$$\nabla^2 T = g/K$$

$$\nabla^2 V = -\rho/\epsilon_0$$

Adjusted your relaxation spreadsheet for the Neuman boundary condtions on the Earth's surface while fixing them at 0 degrees on the sides and bottom. Plot the temperature distribution on the surface of the Earth. Use at least a 30x30 grid with the 1000 degree cell placed at about 8 spreadsheet cells below the surface.

Ferromagnetism revisited



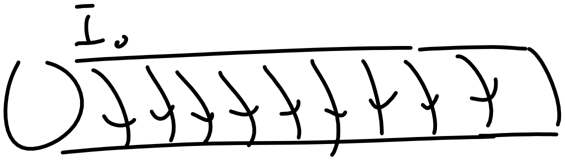
Permanent magnet

Saturation (all domains point in the same direction)

Hysteresis loop

Currie point: temperature above which the material goes from ferromagnetic to what? Curie point heat engine youtube.

An impulsive force can misalign the domains. Demo making a magnetic using the Earth B and striking the iron with a hammer.



Long solenoid filled with iron

$$\vec\nabla \times \underbrace{\left(\frac{\vec B}{\mu_0} - \vec M\right)}_{\vec H} = \vec J_f$$

Amperes's law in $\vec H$

$$\vec\nabla \times \vec H = \vec J_f$$

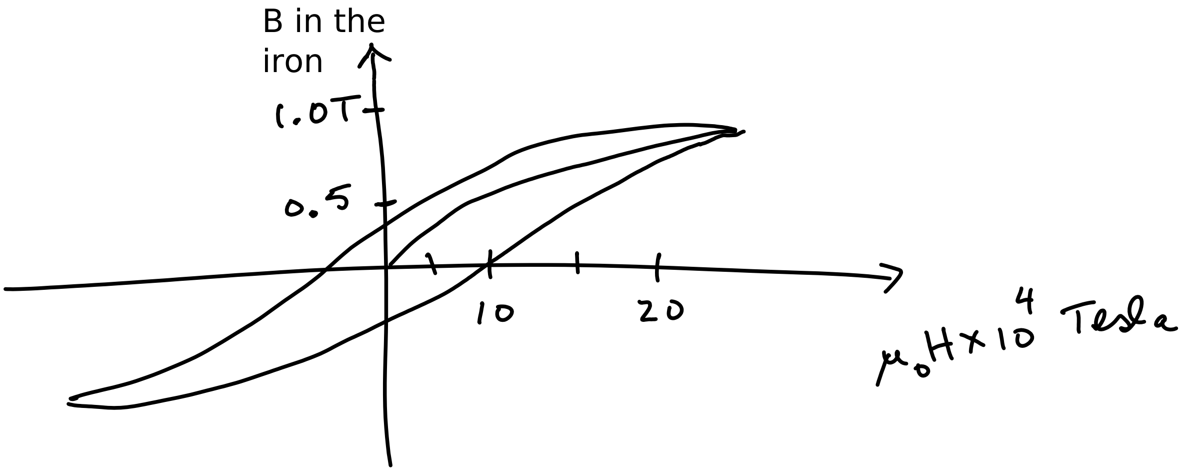
Linear material

$$\vec M \propto \vec H$$

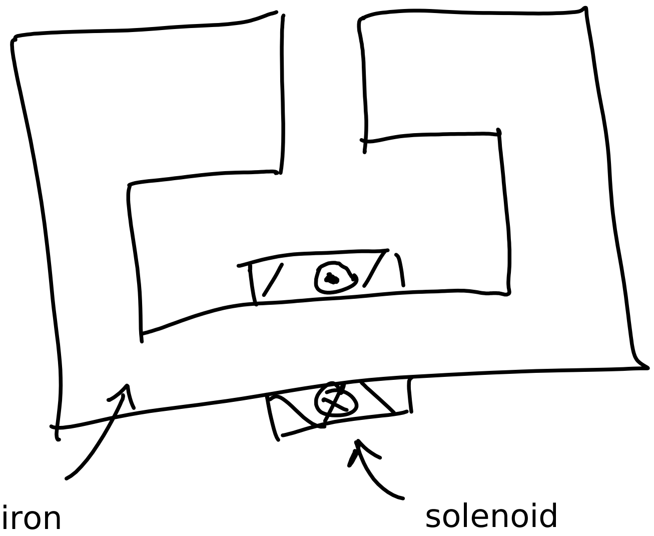
$$HL = NI_0$$

$$\mu_0 H = \mu_0 n I_0 \leftarrow ?$$

turns/length



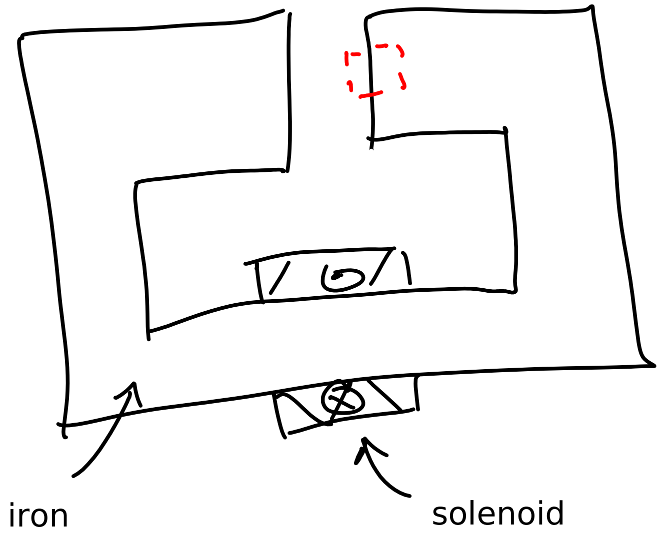
The bound currents in the iron are a thousand times larger than the free currents in the solenoid.



Questions:

-congruous: How do I calculate B in the gap?

As is typical in boundary issues use Maxwell's eqns. in a small interface region.



$\vec{\nabla}\cdot\vec{B} = 0$

$\downarrow$ divergence theorem

$\oint \vec{B}\cdot d\vec{a} = 0$

$\vec{B}\cdot d\vec{a}$ sides

$\vec{B}\cdot d\vec{a}$ end caps $\Rightarrow$ $B_{vac} = B_{iron}$

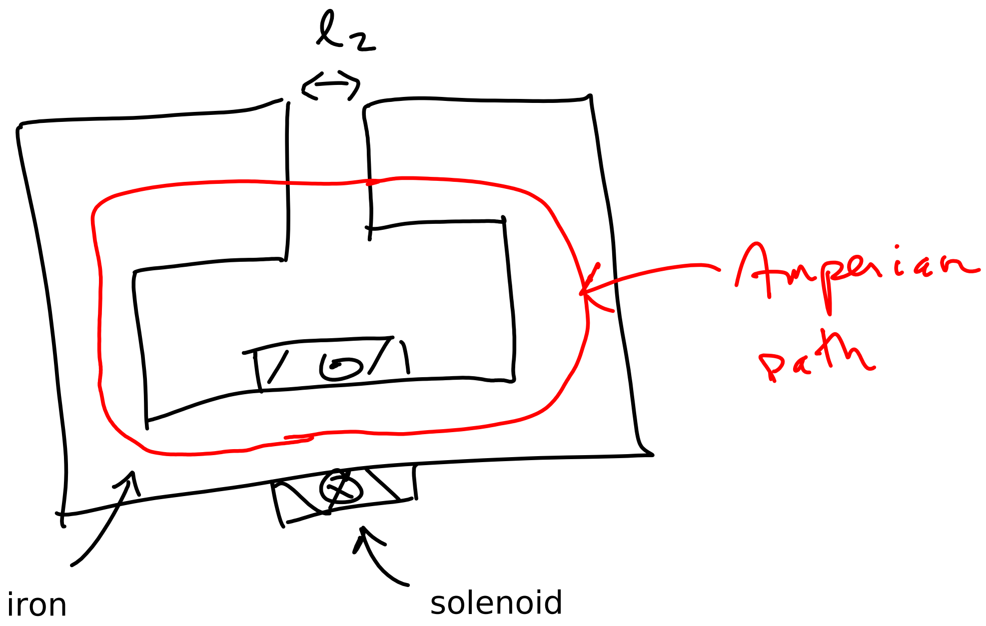
What fundamental eqn do you start with to find B in the gap of this magnet?

Ampere's law: $\vec{\nabla}\times\vec{H} = \vec{J}_f$

$\downarrow$ stokes theorem

$\oint \vec{H}\cdot d\vec{r} = I_f$

What Amperian path?



$$H_{iron} l_1 + H_{air} l_2 = N I_0$$

$$\vec{H} \equiv \left(\frac{\vec{B}}{\mu_0} - \vec{M}\right)$$

$$H_{air} = \frac{B_{air}}{\mu_0} = \frac{B_{iron}}{\mu_0}$$

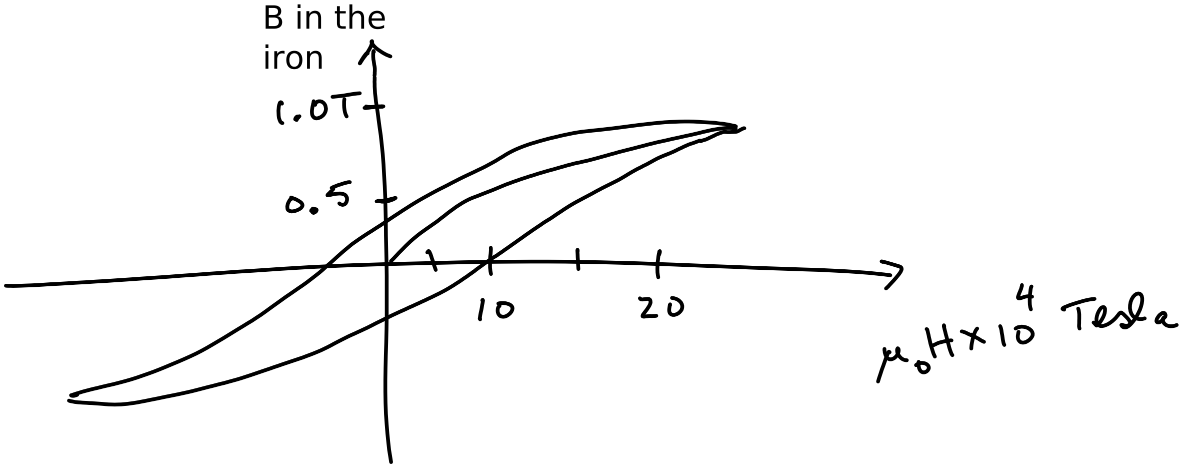
$$H_{iron} l_1 + \frac{B_{iron}}{\mu_0} l_2 = N I_0$$

What is unknown?

We need a relation between B and H in the iron.

informational: What is this relation?

That is the magnetization curve which is non-linear.

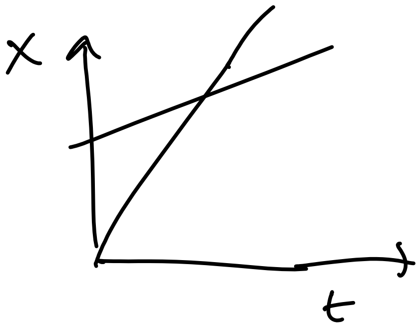


This is the needed other eqn in the two unknowns. Two eqns. in two unknowns yields the answer.

congruous: How do I calculate the solution?

analogous: How did we solve two eqns in two unknows for the wave hitting the moving mirror?

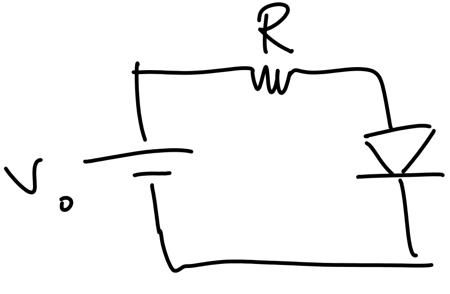
Graphical soln



$$x_{mirror} = L_0 + V_0 t$$

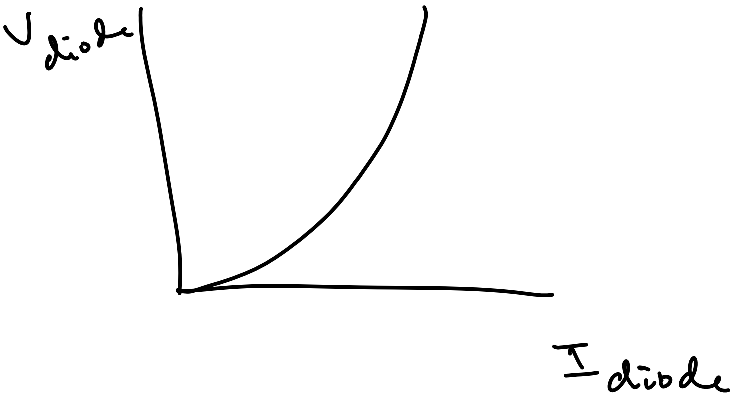
$$x_{crest} = ct$$

analogous: How did we solve for the current in a diode?

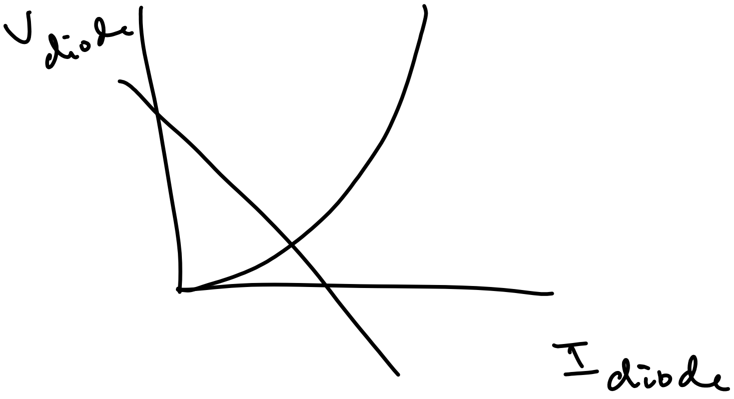


$$V_0 - IR - V_{diode} = 0$$

congruous: How do I calculate the voltage current relation for a diode?



$$V_0 - IR - V_{diode} = 0 \Rightarrow V_{diode} = V_0 - I_{diode}R$$



analogous: How can I use these calculations to calculate B in the vacuum region of the magnet?

$$H_{iron}\, l_1 + \frac{B_{iron}}{\mu_0}\, l_2 = N I_0$$

$$B_{iron} = \frac{\mu_0 N I_0}{l_2} - \mu_0 H_{iron} \frac{l_1}{l_2}$$