

10/6/06

Note Title

10/6/2006

## midterm Review

Roel Snieder available next  
week:      Monday 2-3  
              Tuesday all afternoon

Green Center 240 F

Exam will be 50 minutes

open book

3 out of 4 questions

NO Least squares or  
 $\epsilon$ -values (covered on  
next hw).

## Practice questions

$$A = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} = \begin{pmatrix} \lambda_1^2 & 0 \\ 0 & \lambda_2^2 \end{pmatrix}$$

...

$$a) \quad A^n = \begin{pmatrix} \lambda_1^n & 0 \\ 0 & \lambda_2^n \end{pmatrix}$$

$$b) \quad \ln(I + A)$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \dots$$

Hence

$$\det(I + A) = A - \frac{A^2}{2} + \frac{A^3}{3} - \frac{A^4}{4} \dots$$

$$= \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} - \frac{1}{2} \begin{bmatrix} \lambda_1^2 & 0 \\ 0 & \lambda_2^2 \end{bmatrix} + \frac{1}{3} \begin{bmatrix} \lambda_1^3 & 0 \\ 0 & \lambda_2^3 \end{bmatrix} \dots$$

$$= \begin{bmatrix} \lambda_1 - \frac{1}{2} \lambda_1^2 + \frac{1}{3} \lambda_1^3 + \dots & 0 \\ 0 & \lambda_2 - \frac{1}{2} \lambda_2^2 + \frac{1}{3} \lambda_2^3 - \dots \end{bmatrix}$$

$$= \begin{bmatrix} \det(1 + \lambda_1) & 0 \\ 0 & \det(1 + \lambda_2) \end{bmatrix}$$

$$\bullet \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}$$

$$= \lim_{x \rightarrow 0} \frac{1 - (1 - x^2/2 + x^4/24 \dots)}{x^2}$$

$$= \lim_{x \rightarrow 0} \frac{1}{2} - \frac{x^2}{24} \dots$$

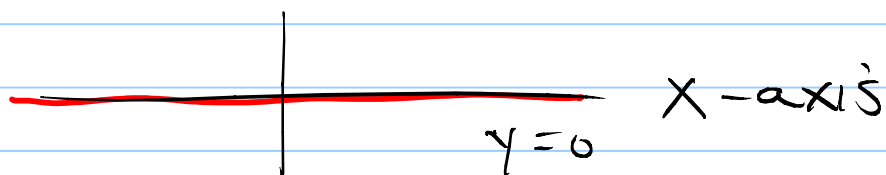
$$= \frac{1}{2}$$

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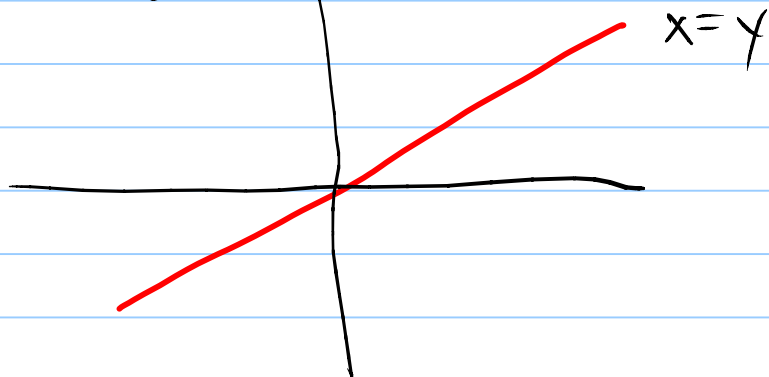
Null space of Column Space

$$\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$$

Column Space =  
span  $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$



$$\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow x=y$$



$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

all of  $\mathbb{R}^3$  is null space

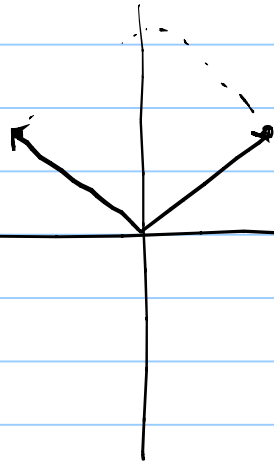
column space =  $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ .

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Box problem

$$\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$



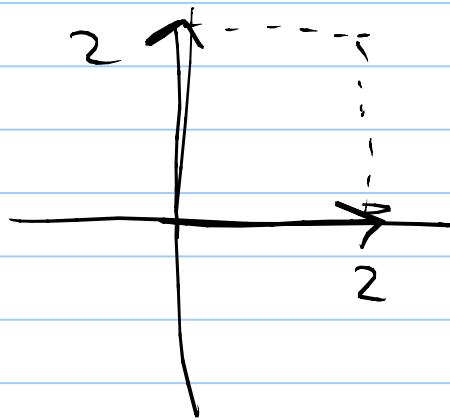
rotation.

NB  $\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

So  $\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$  is an orthogonal matrix

$$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

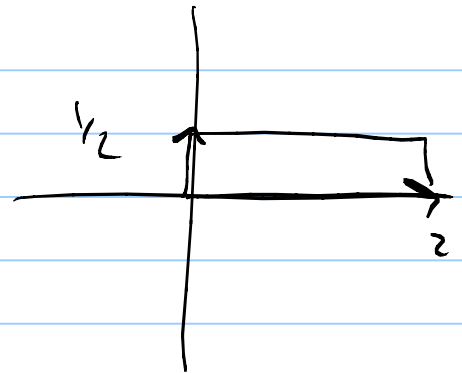
$$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$



stretch

$$\begin{bmatrix} 2 & 0 \\ 0 & 1/2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 \\ 0 & 1/2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1/2 \end{bmatrix}$$

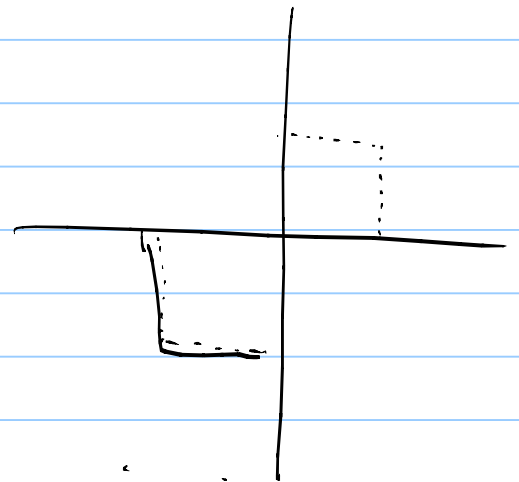


shrink in 1 dim.

Stretch in the other

$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

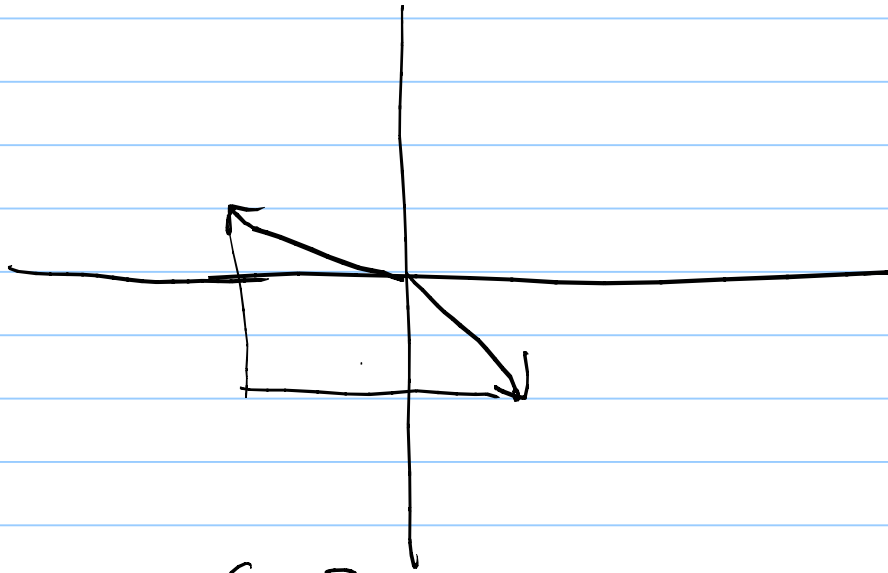


reflection about origin

So far all of these transf.  
Have preserved the angle  
between the vectors

$$\begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$



$$\underbrace{[-2, 1]} \cdot \begin{bmatrix} 1 \\ -2 \end{bmatrix} = -2 - 2 = -4$$

$$\| \cdot \| = \sqrt{2^2 + 1^2} = \sqrt{5}$$

$$\cos \theta = \frac{-4}{\sqrt{5}}$$



$$\frac{x}{(1-x)^2} = x + 2x^2 + 3x^3 + 4x^4 + \dots$$

$$= \sum_{k=1}^{\infty} kx^k$$

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$$\text{Let } P = \sum_{k=m}^{\infty} x^k$$

$$= x^m + x^{m+1} + \dots + x^{\infty}$$

$$\text{Let } S_n = 1 + x + x^2 + \dots + x^n$$

$$\text{So } P = S_n - \underbrace{[1 + x + x^2 + \dots + x^{m-1}]}_{S_{m-1}}$$

$$S_n = \frac{1 - x^{n+1}}{1 - x}$$

$$S_{m-1} = \frac{1 - x^m}{1 - x}$$

$$\Rightarrow P = \frac{1 - x^{2+1}}{1 - x} - \frac{1 - x^3}{1 - x}$$

$$= \frac{x^3 - x^{2+1}}{1 - x}$$

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$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \alpha \\ \beta \\ \alpha \end{pmatrix}$$

case 1)  $\alpha = 1$   $\beta = \alpha = 0$

$$\begin{aligned} x + y &= 1 & x &= 1 \\ y &= 0 & y &= 0 \\ 2y &= 0 \end{aligned}$$

So

$$1 \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + 0 \cdot \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

case 2  $\alpha = \beta = 0$   $\delta = 1$

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \Rightarrow$$

$$\left. \begin{array}{l} x + y = 0 \\ y = 0 \end{array} \right\} x = y = 0$$

$$2y = -1 \quad \text{impossible}$$

therefore RHS not in  
column space.

$$A = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \quad B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$C = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \quad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

x	A	B	C	I
A	I	C	B	A
B	C	I	A	B
C	B	A	I	C
I	A	B	C	I

$$\vec{y} = T \vec{x}$$

$\vec{x}, \vec{y}$  vectors

$T$  matrix

given  $x$  &  $y$  is  $T$  determined uniquely?

$$y_1 = T_{11}x_1 + T_{12}x_2 + T_{13}x_3$$

$$y_2 = T_{21}x_1 + T_{22}x_2 + T_{23}x_3$$

$$y_3 = T_{31}x_1 + T_{32}x_2 + T_{33}x_3$$

suppose

$$T = \begin{bmatrix} T_{11} & & \\ & T_{22} & \\ & & T_{33} \end{bmatrix}$$

Then

$$T_{11} = y_1/x_1$$

$$T_{22} = y_2/x_2$$

$$T_{33} = y_3/x_3$$

This  $T_1$  satisfies  $\vec{y} = T\vec{x}$

But how about

$$T_2 = \begin{bmatrix} & & T_{13} \\ & T_{22} & \\ T_{31} & & \end{bmatrix}$$

then

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} & & T_{13} \\ & T_{22} & \\ T_{31} & & \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\Rightarrow T_{13} = y_1 / x_3$$

$$T_{22} = y_2 / x_2$$

$$T_{31} = y_3 / x_1$$

So both  $T_1$  &  $T_2$  satisfy

$$\vec{y} = T \vec{x}$$

The consequence of this is that division by vectors is not defined in general