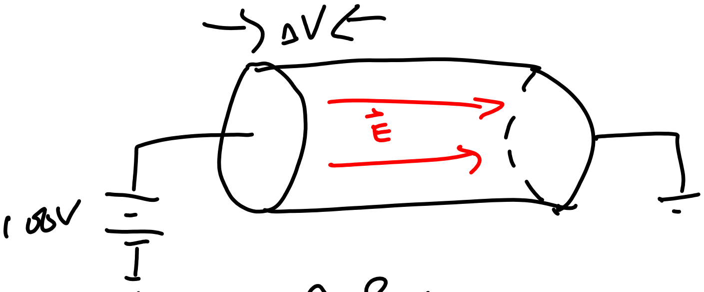
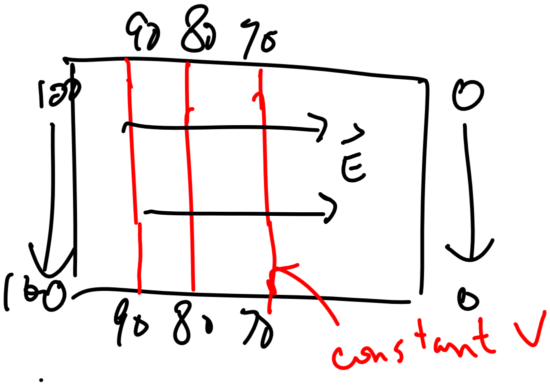
Lecture 32 Homework due April 16

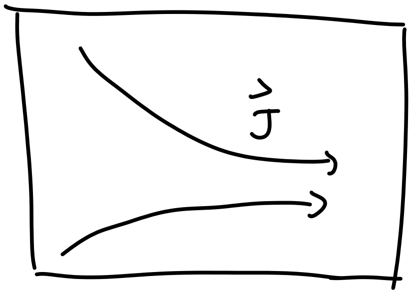
Here is the boundary condition on a copper wire

$$\Delta V = -\int \vec{E}\cdot d\vec{r} .$$





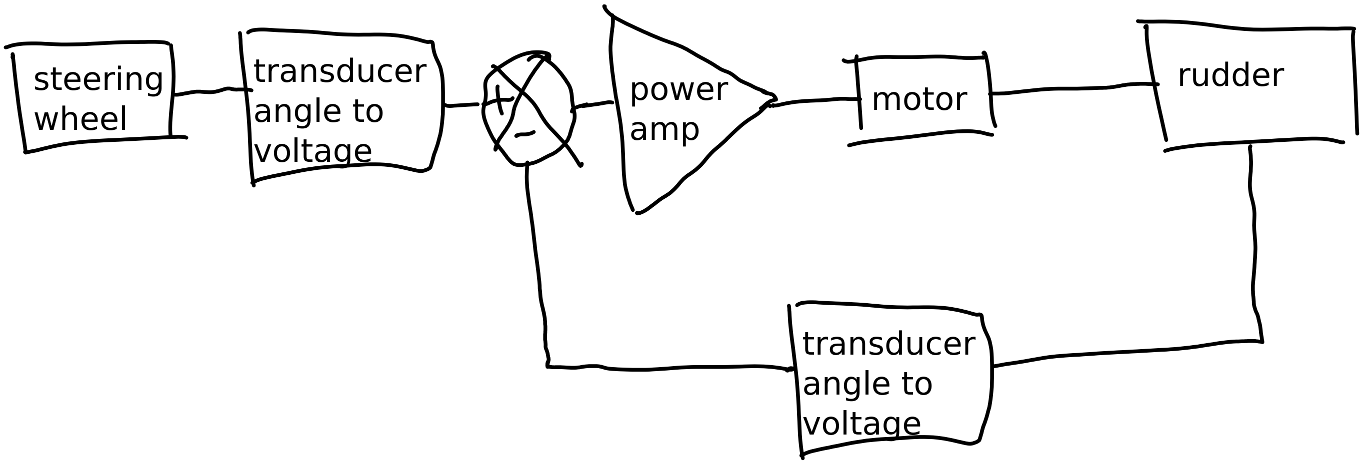
Homework problem 1.) Design the boundary values to focus the current in the center of your boundary as the current travels in an Ohmic material from the left to the right side. Use the relaxation method and include a printout of your spreadsheet. How would you determine how much thermal power (lecture Feb 24) is delivered at the "focal point?"



Homework problem 2.) At what frequency would the feedback signal go from complete negative to complete positive feedback if the only time delay is from the rudder back to the amplifier and is of magnitude tau?

I want to illustrate how retardation effects feedback.

Block diagram of power steering (airplane rudder etc)



$V_{wheel} - V_{rudder}$ goes into amp

In steady state this voltage difference is zero.

Now the temperature increases making the amp reducing the power out of the amp and there for the angle of the rudder.

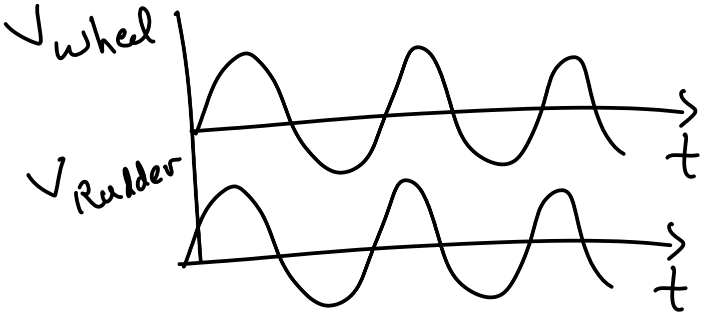
$$V_{wheel} - V_{rudder} > 0$$

This positive error causes the amp to put out more power thereby moving the rudder to a position where this voltage difference is zero.

The same thing happens if you turn the wheel to a larger angle: the difference in voltage causes the amp to put out more power and move the rudder until this voltage difference is zero.

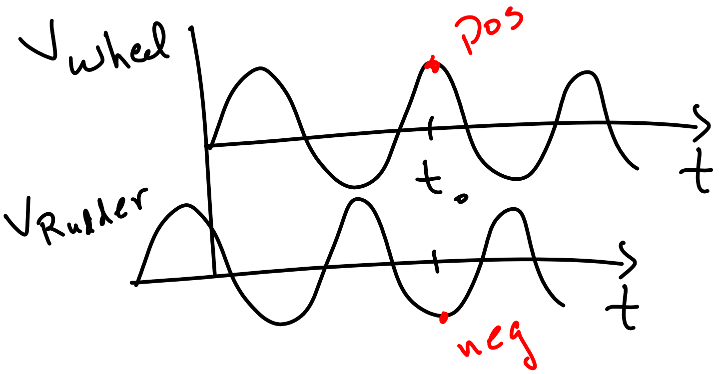
Now turn the vehicle left and right and left and right etc.

There will be a time delay due to the inertia of the rudder, delay in circuits etc



This retarded time effect is called lag and lead time in control theory.

The retardation time can be large enough for this



$$\underset{\text{pos}}{\underset{\downarrow}{V_{wheel}}} - \underset{\text{neg}}{\underset{\downarrow}{V_{Rudder}}} = \text{pos feedback}$$

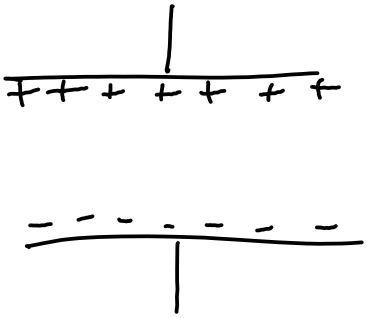
This positive feedback generates oscillation of the system.

$$V_{wheel} - V_{Rudder}\Big|_{t_0}$$

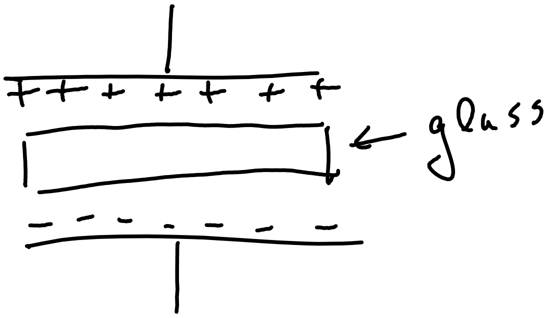
is positive even though the wheel angle and rudder angle are the same. This positive voltage then drives the rudder away when it should do nothing.

These feedback effects occur in you body both chemically and mechanically (e.g. when you try to grab something. Feedback from you eyes helps get you fingers on that object.

Fields in matter



fixed charge Q_0



fixed charge Q_0

Questions based on a model:

-congruous: How do I calculate P in the glass?

-causal/creative: What are the assumptions of the model?

-informational: Why choose fixed charge rather than fixed voltage?

-informational: What is the relation between capacitance, voltage, and charge? $C=Q/V$

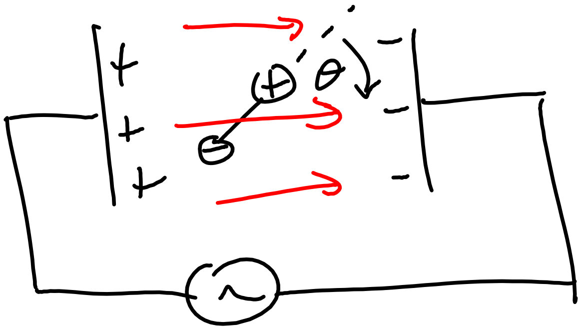
-analogy: How is this mathematically similar to $PV = n RT$? Note V is volume here.

Assume a linear material: $\vec{P} = \epsilon_0 \chi_e \vec{E}$

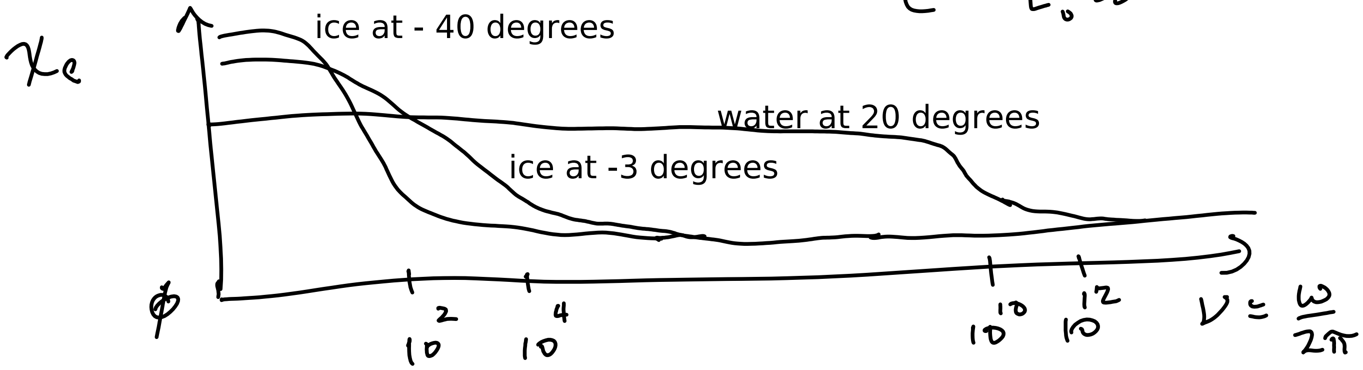
Assume each dipole is induced and not permanent. $\vec{p} = \alpha \vec{E}$

Review the water molecule example

incongruous: Doesn't the susceptibility vary when the dipole is in an oscillating field?



$E = E_0 \cos \omega t$



Even though the field changes the relation is satisfied for a constant χ_e

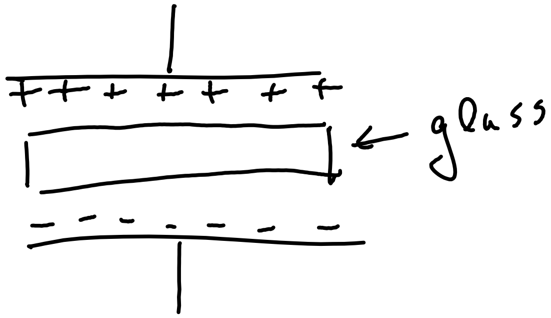
$P = \epsilon_0 \chi_e E_0 e^{i\omega t}$

The dipoles align with the field even though the field changes direction with time. Only when the inertia of the dipole prevents the dipole from rotating does this model fail.

Assumptions:

$$\vec{P} = \epsilon_0 \chi_e \vec{E}$$

- -No permanent dipole moments
- -Uniform field
- -no non-linear terms



fixed charge Q_0

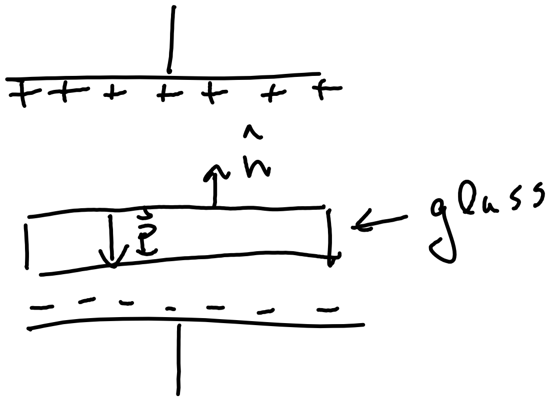
Model: find the free and bound charge densities then throw away the material when calculating the electric field.

$$E = \frac{\sigma_f - \sigma_b}{\epsilon_0}$$

when $\sigma_b = 0$ we get the expected field in the cap.

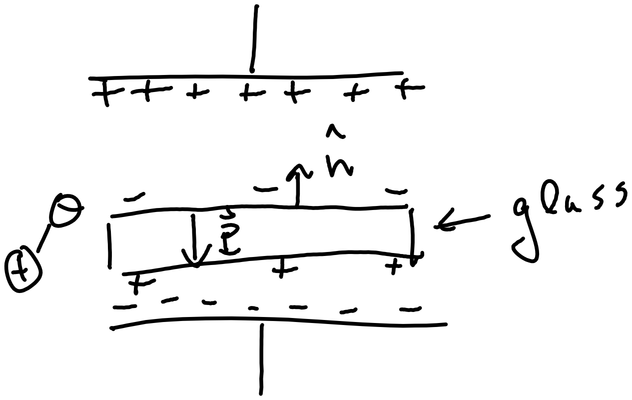
congruous: How do I calculate the bound surface charge?

$$\sigma_b = \vec{P} \cdot \hat{n}$$



fixed charge Q_0

$$E = \frac{\sigma_f - \epsilon_0 \chi_e E}{\epsilon_0}$$



fixed charge Q_0

Questions

causal/creative: What E should we use in the susceptibility term?

$$E_{tot} = \frac{\sigma_f - \epsilon_0 \chi_e E_{tot}}{\epsilon_0}$$

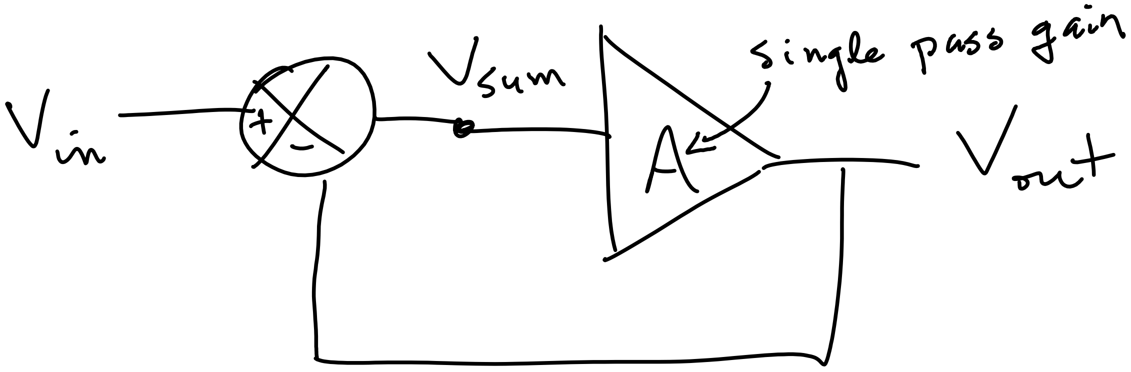
$$E_{tot} + \chi_e E_{tot} = \frac{\sigma_f}{\epsilon_0} \Rightarrow E_{tot} = \frac{\sigma_f}{\epsilon_0} \frac{1}{1+\chi_e} = \frac{E_{vac}}{K}$$

$\nearrow$ dielectric constant

Questions:

analogous: Doen't the math look just like the amplifier problem?

Block diagram of an amplifier with feedback



$$V_{sum} = V_{in} - V_{out}$$

$$V_{out} = A\,V_{sum} = A(V_{in} - V_{out})$$

$$V_{out} = A V_{in} - A V_{out} \; ; \; V_{out} = A V_{in} \frac{1}{1+A}$$

Compare with

$$E_{tot} = \frac{\sigma_f}{\epsilon_0} - \chi_e E_{tot}$$

Perturbative interpretation: 1.) signal get single pass amplified to yield $V_{01} = AV_{in}$

2.) that is fed back to get neg amplified to yield $V_{02} = -AV_{01}$

3.) that is fed back to get neg amplified to yield $V_{03} = -AV_{02}$

$$V_{out}^{tot} = V_{01} - V_{02} + \cdots -\cdot$$

$$E_{tot} = \frac{\sigma_f}{\epsilon_0}\frac{1}{1+\chi_e} = E_0\frac{1}{1+\chi_e}$$

Perturbative interpretation: 1.) Without the glass the field is E_0

2.) that field polarizes the glass reducing the field by $\chi_e E_0$

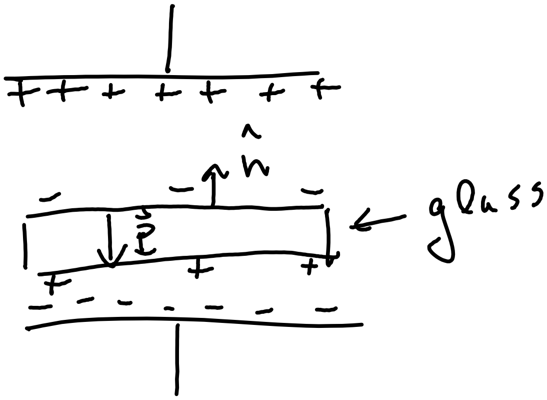
3.) this field again polarizes the glass reducing E by $\chi_e^2 E_0$

$$E_{tot} = E_0(1 - \chi_e - \chi_e^2 - \dots)$$

Return to our problem

$$E_{tot} = \frac{\sigma_f}{\epsilon_0}\frac{1}{1+\chi_e} = \frac{E_{vac}}{K}$$

dielectric constant



fixed charge Q_0

Defns: $K = \frac{\epsilon}{\epsilon_0} = 1+\chi_e$

permittivity of free space (look up permittivity on wikipedia)

congruous: What have we not calculated given the material property K?
Have we calculated everything we need to understand the physics?

We now know $E_{tot} = \frac{\sigma_f}{\epsilon_0}\frac{1}{1+\chi_e} = \frac{E_{vac}}{K}$

The value of K tells us how much E is reduced in the glass. What's K for a conductor?

$$\sigma_b = |\vec{P}\cdot\hat{n}| = \epsilon_0 \chi_e E_{tot} = \epsilon_0 \chi_e \frac{\sigma_f}{\epsilon_0(1+\chi_e)}$$

This example completes what we know from this model.

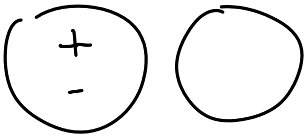
Questions:

congruous: How do I calculate the capacitance from this model?

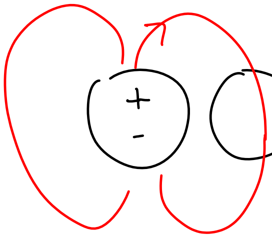
causal/creative: Nothing we have done uses the atomic nature of matter. How is that included? This is one main goals of this physics program is to be able to relate macroscopic phenomina to the microscopic world and vise versa.

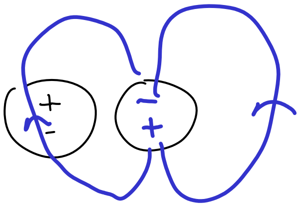
modifying: What happens if the cap is at constant voltage?

modifying: What would happen if the charge aligned in the opposite direction to the applied field?



A collision initially generates a dipole in on atom.



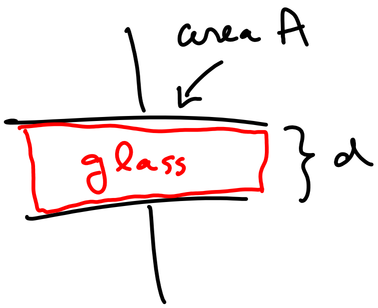


The dipole in the second atom amplifies the dipole in the first atom.

This is an example of positive feedback. The atoms don't "blow" up however. Mathematically this is like summing 1 + .1 + .01 + .001 + . . . This sum does NOT go to infinity so the positive feedback in this case leads to a stable relationship between the atoms.

incongruous: How can this be a valid model of the electric field of glass where the atoms are moving all the time?

Calculation of the capacitance.



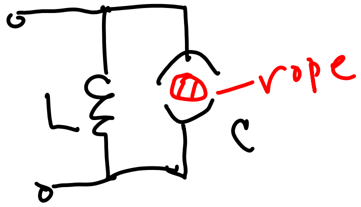
$$C = \frac{Q}{|\Delta V|} = \frac{\sigma A}{E d}$$

what σ? This is the free charge since that is what we can control on a capacitor.

what E? This is the electric field in the glass not in vacuum.

$$C = \frac{\sigma_f A}{\frac{\sigma_f}{\epsilon_0} d \frac{1}{1+\chi_e}} = \frac{\epsilon_0 A}{d} \underbrace{(1+\chi_e)}_{k} = C_0 k$$

I had a student do a senior design project on measuring dirt in a climbing rope. He was told by an expert to move the rope through a capacitor. The dirt would change the capacitance.
The capacitance can be measured accurately using a resonant circuit.



What are some design issues with this arrangement?

causal/creative: What is the cause of this increase in capacitance?

Look up super capacitor on wikipedia.

causal/creative: Nothing we have done uses the atomic nature of matter. How is that included? This is one main goals of this physics program is to be able to relate macroscopic phenomina to the microscopic world and vise versa.