Maxwell equations (static) Differential + integral

$\nabla \cdot \vec{E} = \rho/\varepsilon_0$ $\quad \vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

$\nabla \cdot \vec{B} = 0$ $\quad \oint \vec{B} \cdot d\vec{A} = 0$

$\rightarrow \nabla \times \vec{E} = 0$ $\quad \int (\nabla \times \vec{E}) \cdot d\vec{A} = 0 \Rightarrow \oint \vec{E} \cdot d\vec{l} = 0$ (voltage)

$\int (\nabla \cdot \vec{E}) dV = \int \rho/\varepsilon_0 \, dV$ $\quad \oint \vec{E} \cdot d\vec{A} = Q_{enc}/\varepsilon_0$

$\nabla \times \vec{B} = \mu_0 \vec{J} \Rightarrow \oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$

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Faraday's law $\quad EMF = -\frac{d\Phi_B}{dt}$

$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ (EMF)

$\int (\nabla \times \vec{E}) \cdot d\vec{A} = -\int_{S(t)} \frac{\partial \vec{B}}{\partial t} \cdot d\vec{A} = \oint \vec{E} \cdot d\vec{l}$ (Leibniz rule)

$\frac{d}{dt} \int_{a(t)}^{b(t)} f(x,t) dx = \frac{db}{dt} f(b,t) - \frac{da}{dt} f(a,t) + \int_a^b \frac{\partial}{\partial t} f(x,t) dx$

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3D

$\oint \vec{E} \cdot d\vec{l} = -\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{A}$

$\frac{d}{dt} \int_{S(t)} \vec{B}(\vec{x},t) \cdot d\vec{A} = \int \left(\frac{\partial \vec{B}}{\partial t} + (\nabla \cdot \vec{B})\vec{v}\right) \cdot d\vec{A} - \oint (\vec{v} \times \vec{B}) \cdot d\vec{l}$ ($\nabla \cdot \vec{B} = 0$)

$-\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{A} = -\frac{d}{dt} \int_{S(t)} \vec{B} \cdot d\vec{A} - \oint (\vec{v} \times \vec{B}) \cdot d\vec{l}$

$\oint \vec{E} \cdot d\vec{l} + \oint (\vec{v} \times \vec{B}) \cdot d\vec{l} = \oint (\vec{E} + \vec{v} \times \vec{B}) \cdot d\vec{l} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{A}$

$= -\frac{d\Phi_B}{dt}$

$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

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$\nabla \cdot \vec{E} = \rho/\varepsilon_0$

$\nabla \cdot \vec{B} = 0$

$\nabla \times \vec{E} = -\partial \vec{B}/\partial t \Rightarrow \oint \vec{E} \cdot d\vec{l} = -\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{A}$

$\nabla \times \vec{B} = \mu_0 \vec{J} \Rightarrow \oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$

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$EMF = -\frac{d\Phi_B}{dt}$

$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$\oint \vec{E} \cdot d\vec{l} = -\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{A}$

$\nabla \times \vec{E} = -i\omega \vec{B}$

$\oint (\vec{E} + \vec{v} \times \vec{B}) \cdot d\vec{l} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{A}$

~~$\oint \vec{E} \cdot d\vec{l} = \frac{d}{dt} \int \vec{B} \cdot d\vec{A}$~~

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