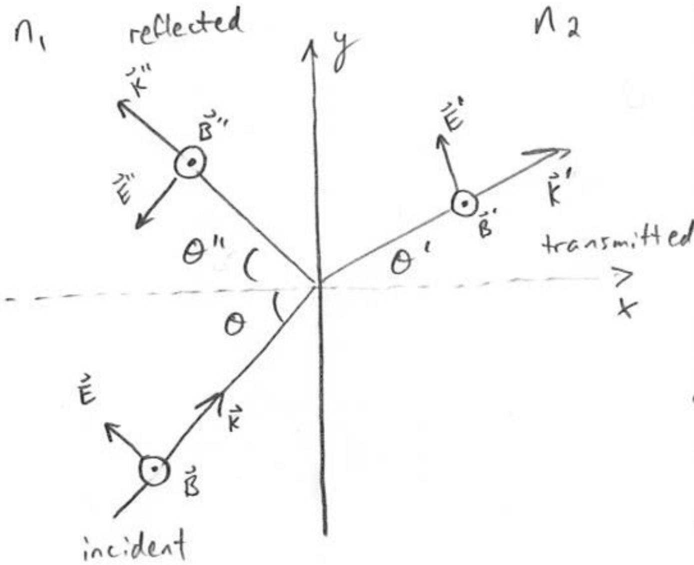
Recitation 2 – Dielectric interfaces

Let's suppose we have TM-polarized light incident on the boundary between two dielectrics with indices n_1 and n_2. The incident light makes some nonzero angle θ with the optical axis. Sketch that situation, including the directions of the k vector, E-field, and B-field for each of the three waves involved.



I'm not yet making any assumptions about θ, θ', and θ'', but we have shown in the past that $\theta = \theta''$ and $n_1 \sin\theta = n_2 \sin\theta'$.

Note that I'm fixing the relative directions of $\vec{K}$, $\bar{E}$, and $\bar{B}$ by using the Poynting vector. We know $\vec{k}$ is in the $\bar{E} \times \vec{B}$ direction.

Given an incident electric field of the form

$$\vec{E}_I(\vec{x}, t) = \vec{E}_0 e^{i(\vec{k}\cdot\vec{x} - \omega t + \delta)}$$

What phase angle δ could we choose to represent an incident E-field that has zero magnitude when $\vec{x}$ and t are zero?

Well, when $\vec{x}$ and t are zero, we get

$$\vec{E}_I = \vec{E}_0 e^{i\delta} \qquad \text{And} \qquad e^{i(0)} = 1, \qquad e^{i(\pi/2)} = i$$

$e^{i\delta}$ is never zero by itself for any real δ, but since we take the real part of these expressions to get the physical fields, and $\mathrm{Re}\{e^{i\pi/2}\} = 0$, a phase angle of $\delta = \pi/2$ would do it.

Write the four field boundary conditions that we can most easily apply to dielectric interface problems. Keep them in a general form; don't adapt them to this specific problem yet.

$D_{1\perp} = D_{2\perp}$, or $\varepsilon_1 E_{1,\perp} = \varepsilon_2 E_{2,\perp}$

$B_{1,\perp} = B_{2,\perp}$

$E_{1,\parallel} = E_{2,\parallel}$

$H_{1,\parallel} = H_{2,\parallel}$, or $B_{1,\parallel}/\mu_1 = B_{2,\parallel}/\mu_2$

These come from our Maxwell equations in matter with the free sources set to zero:

$$\nabla \cdot \vec{D} = 0 \qquad \nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \nabla \times \vec{H} = +\frac{\partial D}{\partial t}$$

Explain why we don't directly use the boundary condition that reads $E_{1,\perp} - E_{2,\perp} = {}^{\sigma}/_{\varepsilon_0}$.

The σ in that equation includes a bound charge that comes about through polarization. Since the polarization of the material depends on the fields that we're trying to solve for, that boundary condition ends up being inconvenient at best.