PH311 exam 1: 10/10/07

Do any four (and **only** four) of the following questions.

1: 25 points

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

(7) Compute the inverse of this matrix.

(7) What is the characteristic polynomial of A?

(7) What are the eigenvalues?

(4) Is this matrix diagonalizable?

2: 25 points Here is a simple model of radioactive decay. The model predicts that during a time interval T, a certain fraction P of a substance will decay and, hence, a fraction $(1-P)$ will not decay. Show that if we start with A undecayed nuclei, then the number of nuclei that decay on average during n periods of time is given by a geometric series:

$$PA + PA(1-P) + PA(1-P)2 + ... + PA(1-P)^{(n-1)}$$

Verify that that in the limit that n goes to infinity this sum equals A. Explain the physical significance of this result.

3: 25 points Without using a calculator to compute $\sin(.1)$ exactly, estimate the size of the error in the approximation $\sin(x) \approx x$ when $x = .1$ radians. Hint: McClaurin series.

4: 25 points Below are four matrices. One is Hermitian, one is symmetric, one is orthogonal and one is unitary. Which is which? Each part has equal weight.

(a)

$$\begin{pmatrix} 1 & 2i & 3+4i \\ -2i & 5 & 6-7i \\ 3-4i & 6+7i & 8 \end{pmatrix}$$

(b)

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{i}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 \\ 0 & 0 & i \end{pmatrix}$$

(c)

$$\frac{1}{3}\begin{pmatrix} 2 & -2 & 1 \\ 1 & 2 & 2 \\ 2 & 1 & -2 \end{pmatrix}$$

(d)

$$\begin{pmatrix} 66 & 78 & 90 \\ 78 & 93 & 108 \\ 90 & 108 & 126 \end{pmatrix}$$

5: 25 points Compute the amplitude and phase of the following complex numbers. Each part has equal weight.

(a)

$$z = 1 + i\sqrt{3}$$

(b)

$$z = (1 - i)/(1 + i)$$

(c)

$$z = i^5 + i^6$$

(d)

$$z = 1/(1 - i)$$

6: 25 points For the following linear system:

$$\begin{pmatrix} -1 & 2 \\ 1 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

(a) Without actually solving the linear system explain why it cannot have a unique solution.

(b) Find a solution.

(c) Find a vector which spans the null space.

(d) Give an analytic expression for the set of all possible solutions