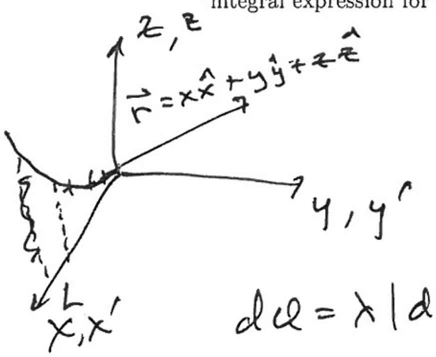
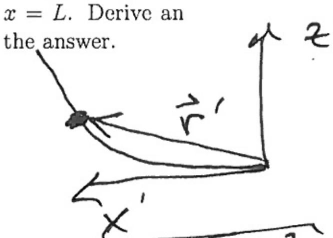
Exam 1   Jan. 20   Name....KEY....

1. Charge is uniformly distributed on a non conducting wire in the shape of a parabola in the x-z plane. The equation determining this shape is $z = x^2$ with charge going from $x = 0$ to $x = L$. Derive an integral expression for $\vec{E}$ at an arbitrary point from which Mathematica will yield the answer.





$$d\vec{E} = \frac{k\,dq}{r^2}\hat{r}$$

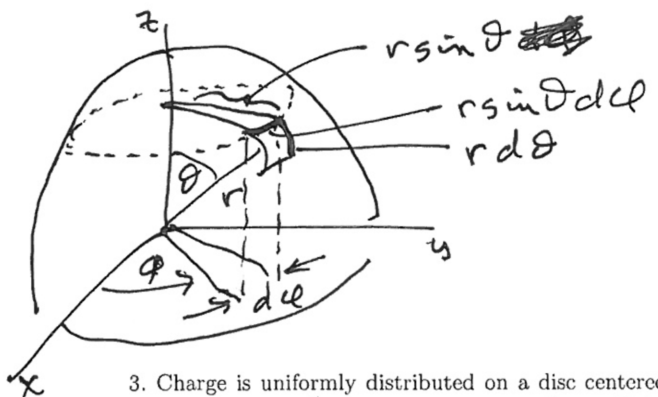
$$\vec{r}' = x'\hat{x} + x'^2\hat{y} + 0\hat{z}$$

$$d\vec{r}' = dx'\hat{x} + 2x'dx'\hat{y} \qquad |d\vec{r}| = dx'\sqrt{1+4x'^2}$$

$$dq = \lambda|d\vec{r}| = \lambda\, dx'\sqrt{1+4x'^2}$$

$$\vec{E} = \int d\vec{E} = \int_0^L \frac{k\lambda\, dx'\sqrt{1+4x'^2}}{r^2}\hat{r}$$

2. Derive an expression for the infinitesimal surface area (or tile) placed on the surface of a sphere of radius R. Write an integral from this with limits to determine the surface area of this sphere.



$$\int_0^{2\pi}\int_0^{\pi} r\sin\theta\, d\varphi\, r\, d\theta = \text{Vol}$$

3. Charge is uniformly distributed on a disc centered in the x-y plane of radius R. Derive an integral expression for $\vec{E}$ at an arbitrary point from which Mathematica will yield the answer. Write on the back of this page.

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