2_8_08

Note Title 2/8/2008

Free particle $V(x)=0$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2}=E\psi$$

$$\psi''+\frac{2mE}{\hbar^2}\psi=0$$

$$\psi''+k^2\psi=0 \qquad k^2=\frac{2mE}{\hbar^2}$$

$$\psi(x)=Ae^{ikx}+Be^{-ikx}$$

$$\Psi(x,t)=Ae^{i(kx-Et/\hbar)}+Be^{-(kx+Et/\hbar)}$$

factor out ak $\quad e^{ik(x-\frac{E}{kh}t)}$

$$\frac{E}{k\hbar}=\frac{\hbar k}{2m}$$

$$\Psi(x,t)=A\underbrace{e^{ik(x-\frac{k\hbar}{2m}t)}}+B\underbrace{e^{-ik(x-\frac{k\hbar}{2mE})}}$$

plane waves moving to left & right

$$x\pm vt$$

$$v=\frac{\hbar k}{2m}$$

$$k = \pm \frac{\sqrt{2mE}}{\hbar}$$

$k > 0$ traveling to right

< 0 left

$$\boxed{\text{deBroglie} \qquad p = \hbar k}$$

$$v = \frac{\hbar k}{2m} = \frac{p}{2m} = \frac{1}{2} v_{classical}$$

Not normalizable. form a Linear combination

$$\Psi(x,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \phi(k)\, e^{i(kx - \hbar k^2/2m\, t)}\, dk$$

for appropriate choice of ϕ this can be normalized

Now, as usual, we will be given $\Psi(x, t=0)$ and asked to find $\Psi(x,t)$.

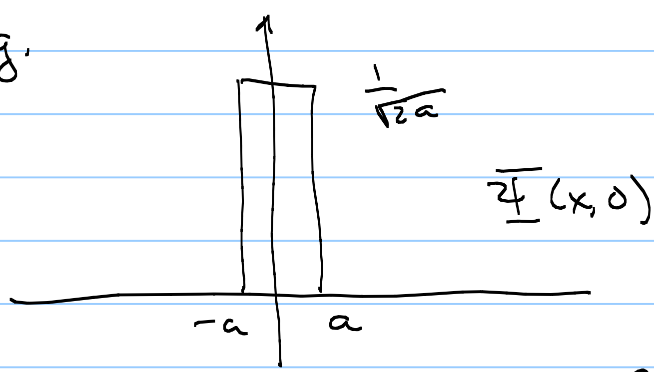
$$\Psi(x,0) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \phi(k)\, e^{ikx}\, dk$$

Hence $$\boxed{\phi(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \Psi(x,0)\, e^{-ikx}\, dx}$$

Recall from 311

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(k) e^{ikx}\, dk \Leftrightarrow F(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)\, e^{-ikx}\, dx$$

Eg.



$$\phi(k) = \frac{1}{\sqrt{2\pi}} \frac{1}{\sqrt{2a}} \underbrace{\int_{-a}^{a} e^{-ikx}\, dx}_{\frac{1}{-ik} e^{-ikx} \Big|_{-a}^{a}}$$

$$\frac{1}{2}\frac{1}{\sqrt{\pi a}}\frac{i}{k}\underbrace{\left[e^{-ika}-e^{+ika}\right]}_{-2i\sin ka}$$

$$=\frac{1}{\sqrt{\pi a}}\frac{\sin ka}{k}$$

So $$\Psi(x,t)=\frac{1}{\pi}\frac{1}{\sqrt{2a}}\underbrace{\int_{-\infty}^{\infty}\frac{\sin ka}{k}e^{-ik\left(x-\frac{\hbar k}{2m}t\right)}dk}_{\text{can't be don analytically}}$$

Standard form of a wave packet

$$\Psi(x,t)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\phi(k)\,e^{i(kx-\omega t)}$$

in our case $\omega=\hbar k^2/2m$

$\omega(k)$ is called the dispersion relation

$$\frac{\omega}{k}=v_{phase}\qquad\qquad\frac{d\omega}{dk}=v_{group}$$

$$\omega = \frac{\hbar k^2}{2m}$$

for free particle $\quad \frac{d\omega}{dk} = \frac{k\hbar}{m}$

$$\frac{\omega}{k} = \frac{\hbar k}{2m}$$

$$V_{group} = V_{classical} = 2\,V_{phase}$$