Nonlinear Optics
Homework 4
due Wednesday, 11 Feb 2009

- **Problem 1:**
  The impulse response for a classical damped harmonic oscillator is
  h[t] = Exp[- γ t] Sin[ω0s t] for t > 0, and h[t] = 0 for t<0
  where ω0s is a slightly shifted resonance frequency defined by
  $\omega 0s = \sqrt{\omega 0^2 - \gamma^2}$

  Use transform theorems to calculate the corresponding transfer function to show that you recover what is expected from the linear solution described in Chapter 1. You may check your result by doing the transform in *Mathematica*, but I want the work to be done analytically. You may find the pdf of Fourier Transform theorems useful (see website).

- **Problem 2: Doubling of chirped pulses**
  a. By representing a linearly-chirped Gaussian pulse in the time domain (see notes), calculate the ideal frequency doubling signal (no phase mismatch - output E-field is just the square of the input E-field).
  b. Next transform to the spectral domain (not from scratch - scale the expression for the transform of the chirped pulse from ω- to t-space). From this expression, determine the spectral width and calculate the duration of the doubled pulse if the remaining chirp was removed.
  c. Compare this result to what you get if you just assume that the temporal profile of the input pulse looks just like the input spectrum.

- **Problem 3: opposite chirp sum frequency mixing of pulses**
  Instead of doubling, we do sum frequency mixing of two pulses with opposite chirp.
  a. Show that the output pulse is transform-limited, with a duration equal to $1/\sqrt{2}$ times the duration of the chirped input pulse. This is one way to efficiently make narrowband pulses from ultrashort pulses.
  b. Now introduce a relative time delay T between the pulses (the math works out easier if you shift them in opposite directions by T/2. Describe the nature of the output pulse in this case as a function of the delay T: temporal shape and width, peak amplitude, instantaneous frequency characteristics.

- **Problem 4: OPA of short pulses**
  In an OPA the gain in the undepleted limit is exponentially dependent on the pump pulse intensity. In practice, phase matching limits the output pulse duration, but neglecting this effect, estimate the output pulse duration if the peak parametric gain is 10^6. Assume the pump pulse has a Gaussian temporal profile. Hint: you can expand the pump pulse around its peak to get a simpler form.