Nonlinear Optics
Homework 2
due Wednesday, 28 Jan 2009

- **Problem 1:**
  Boyd problem 2.1. For this problem, assume that "optimum focusing" means that the length of the crystal equals the confocal parameter (2 x the Rayleigh range) of the lowest frequency beam. You don't need to account for focusing phase mismatch (we'll treat this later).

- **Problem 2:**
  The nonlinear crystal used in the OPA paper is $LiIO_3$ (lithium iodate). The Sellmeier equations for $LiIO_3$ are given in the datasheet posted.
  a. Using the Phase_match_KDP.nb notebook as a template, code the dispersion equations for $LiIO_3$ and calculate the variation of n_e with angle θ to the z-axis.
  b. Calculate the optimum phase matching angle for doubling 3000nm light with Type I and Type II. What is the shortest input wavelength that can be phasematched?
  c. For both types, plot the harmonic yield (intensity) as a function of incident angle for a few thicknesses. This shows the angular acceptance.
  d. For both types, plot the harmonic yield as a function of input wavelength for three thicknesses. This shows the phase matching bandwidth.
  e. Now calculate the phase matching angle for the OPA described in the paper (pump at 800nm, idler at 4microns. Make the plot shown in Figure 1 of the paper showing the group velocity matching vs for this crystal.

- **Problem 3:**
  Following the calculations done for second harmonic generation shown in the notebook mixing solutions.nb, numerically solve for the intensity vs. propagation length for sum frequency mixing.
  a) First start wih the nonlinear coupled equations 2.2.11, 2.2.13, and 2.2.14 and redefine the fields scaled to the total intensity $I_{tot} = I_1 + I_2 + I_3$, along the lines of what is shown in 2.6.13, but expressing the fields in terms of a_i instead of $u_i\, e^{i\,\phi_i}$(see my notes). Unlike the case for SHG, there will be some residual frequency dependence to the scaling factor. If you choose your normalization factor to include the frequency ω_3, then the expressions for E_1 and E_2 will have an extra factor of e.g. ω_1/ω_3.

  b) Construct the numerical solution using NDSolve[ ] as shown in my example.
  Find input conditions to generate plots like Figures 2.4.2, 2.4.3 and 2.5.2.

  c) Note that the case of difference frequency mixing (DFM) is also solved here - it is just a matter of what input waves are there. Show an example for DFM, illustrating an OPA (strong pump at ω_3 and a weak seed at ω_1.

  d) For sum-frequency generation, determine conditions that allow complete conversion of the two inputs to ω_3 without any back conversion.