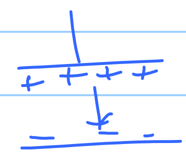
$$V = XYZ = (A + 0x)(A' + 0z)(A'' + By)$$

$$V = \underbrace{AA'A''} + \underbrace{AA'B}\,y$$

$$V = a + by$$

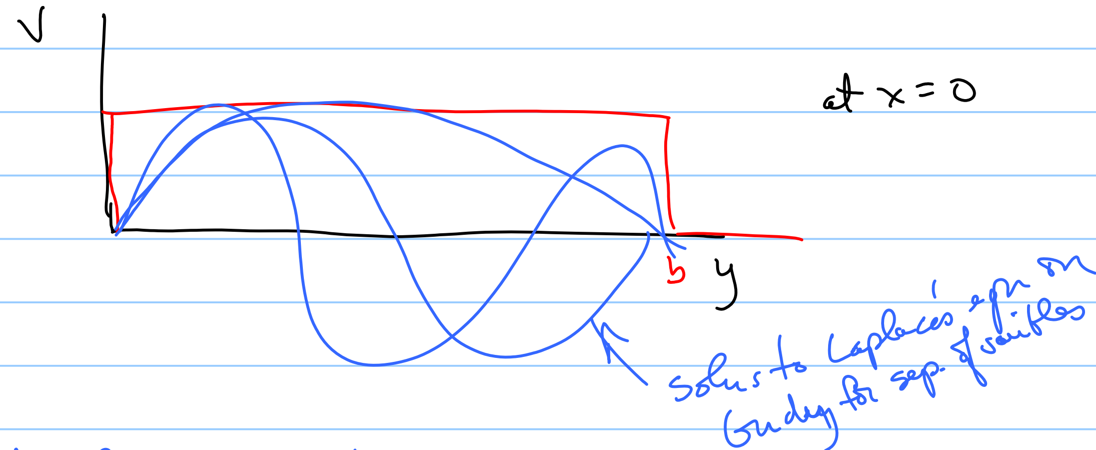
$$V = Ay + B$$

$$\vec{E} = -\vec{\nabla} V = -A\hat{y} \quad \text{const}$$



Back to our 1st problem

$$V = XYZ = \left(Ge^{kx} + He^{-kx}\right)\sin ky \;;\; k = \frac{n\pi}{b}$$



Laplaces eqn obeys superposition ⇒ sum of solns is a soln

$$V = \sum_{i=1}^{N} A_i V_i$$ also a soln also a Fourier series

$$V = \sum_{n=1}^{\infty}\left(A_n e^{-\frac{n\pi x}{b}} + B_n e^{n\pi x/b}\right)\sin\left(\frac{n\pi y}{b}\right)$$ n is integer

A_n & B_n are determined by bndry cond

$$V(x=0) = V_1 = \sum_1 (A_n + B_n) \sin\left(\frac{n\pi y}{b}\right)$$

multiply by $\sin \frac{m\pi y}{b}$ & integrate from $0 \to b$

$$V_1 \int_0^b \sin\left(\frac{m\pi y}{b}\right) dy = \sum_{n} (A_n + B_n) \underbrace{\int_0^b \sin\left(\frac{n\pi y}{b}\right) \sin\left(\frac{m\pi y}{b}\right) dy}$$

$$V_1 \left[-\frac{b}{m\pi} \cos\left(\frac{m\pi y}{b}\right) \right]_0^b = \sum_n (A_n + B_n) \frac{b}{2} \delta_{nm}$$

$$-\frac{bV_1}{m\pi} \left(\cos(m\pi) - 1 \right) = (A_m + B_m) \frac{b}{2}$$

$$\boxed{\begin{array}{lcl} m \text{ odd} & \frac{2bV_1}{m\pi} = (A_m + B_m) \frac{b}{2} \\ m \text{ even} & 0 = (A_m + B_m) \frac{b}{2} \end{array}}$$

1 eqn in 2 unknowns

Other bndy

$$\left\{ V(a) = V_2 = \sum_n \left(A_n e^{-\frac{n\pi a}{b}} + B_n e^{\frac{n\pi a}{b}} \right) \sin\left(\frac{n\pi y}{b}\right) \right\} \times \sin\frac{m\pi y}{b} dy$$

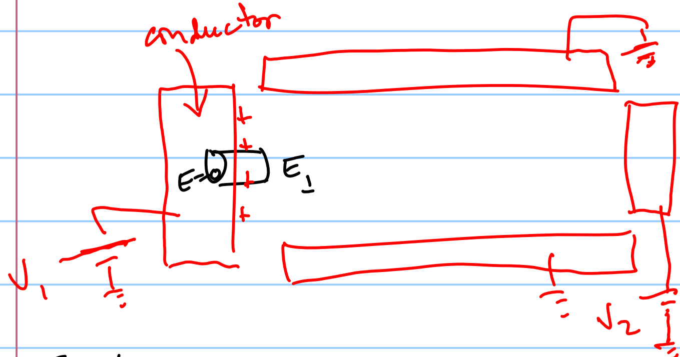
$$\Rightarrow \boxed{A_n e^{-n\pi a/b} + B_n e^{n\pi a/b} = \begin{cases} \frac{4V_2}{n\pi} & n = \text{odd} \\ 0 & n = \text{even} \end{cases}}$$

2 eqns in A_n & B_n

$$A_n = \frac{4}{n\pi}\left(\frac{V_1 - V_2 e^{-n\pi a/b}}{1 - e^{-2n\pi a/b}}\right) \qquad A_n = -B_n$$

final soln

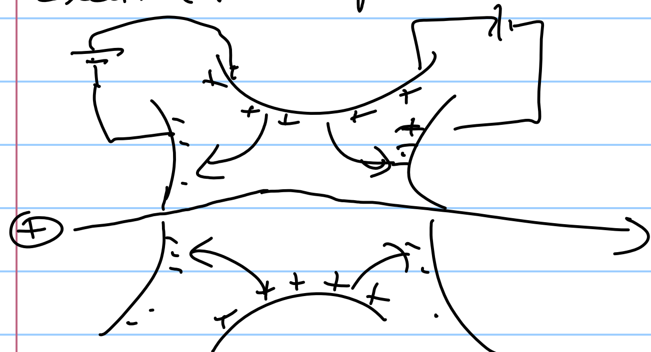
$$V = \sum_{n=\text{odd}}^{\infty}\left(A_n e^{-\frac{n\pi x}{b}} + B_n e^{\frac{n\pi x}{b}}\right)\sin\left(\frac{n\pi y}{b}\right)$$



$$\overset{0}{E_{in}}A + E_{out}\cancel{A} = \frac{Q_{enc}}{\epsilon_0}\cancel{A}$$

$$\frac{\partial V_{above}}{\partial n} - \frac{\partial V_{below}}{\partial n} = -\frac{\sigma}{\epsilon_0}$$

Electron microscope



$$\vec{F} = q\vec{E}$$

use relaxation method (Excel spreadsheet) to solve for V. Then get σ from $\frac{\partial V}{\partial n}$