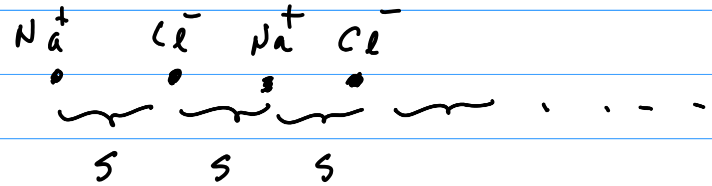
Lecture 14

Shadowitz: 4-2 capacitance, 4-3 electrostatic energy

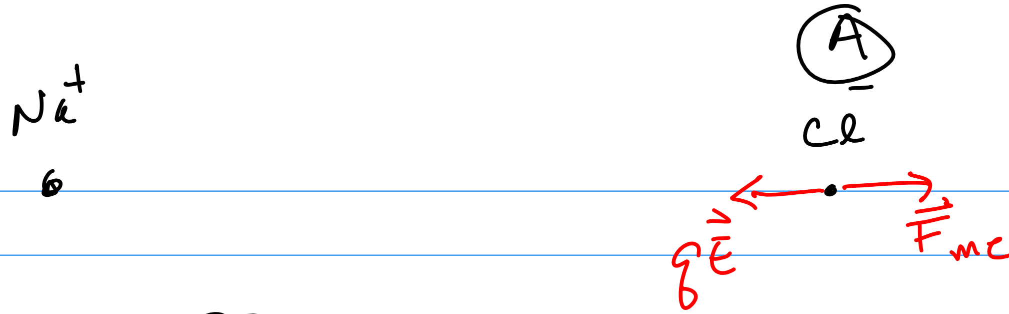
1-D salt crystal

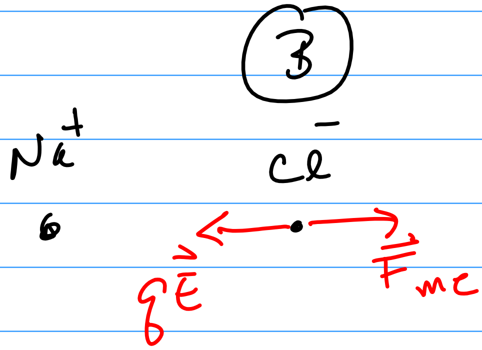


Questions?

How do I calculate the work required to bring in another atom (congruous)?

$W_{net} =$

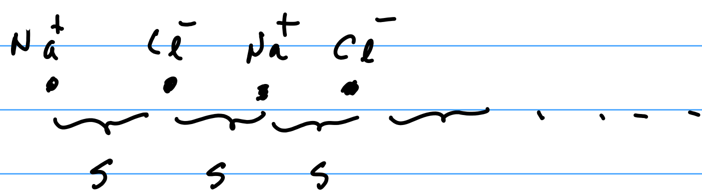




$$W_{me} = -\int_{A}^{B} q\vec{E}\cdot d\vec{r} = q\left(\underbrace{-\int \vec{E}\cdot d\vec{r}}_{\Delta V}\right) = q\left(V_B - V_A\right)$$

$V_A \rightarrow 0$ @ ∞

$$W_{me} = \Delta PE = \frac{q(-q)}{4\pi\epsilon_0 s}$$



$$W_{me} = qV_{total} = q\left[\frac{-q}{4\pi\epsilon_0 s} + \frac{q}{4\pi\epsilon_0 2s} - \frac{q}{4\pi\epsilon_0 3s} \cdots\right]$$

$$= \frac{-q^2}{4\pi\epsilon_0 s}\left[1 - \frac{1}{2} + \frac{1}{3} + \cdots\right] = -\frac{.693\, q^2}{4\pi\epsilon_0 s}$$

x-ray data yields s.

Questions:

Let the next charge be at infinity. Let it go.

$$0 = W_{me} = \Delta(KE + PE) : (KE + PE)_f - (KE + PE)_i = 0$$

$$W_{me} = W_{non\text{-}conservative}$$

$$(KE)_i = 0, \quad (PE)_i = 0$$

How much work do I have to do to assemble the crystal (congrous)?

$$W_1 = 0$$

$$W_2 = \frac{-q^2}{4\pi\epsilon_0} \frac{1}{s}$$

$$W_3 = \frac{-q^2}{4\pi\epsilon_0} \left[\frac{1}{s} - \frac{1}{2s} \right]$$

$$W_4 = \frac{-q^2}{4\pi\epsilon_0} \left[\frac{1}{s} - \frac{1}{2s} + \frac{1}{3s} \right]$$

Questions

$$dW_{me} = V\,dq$$

Voltage at the point where dq is placed due to the charges already present but NOT the charges yet to be brought in.

Questions:

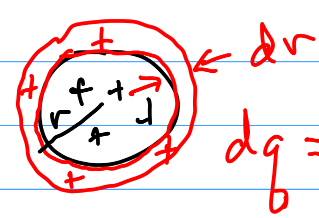
How do you calculate this in a simple example (congrous)?

How much work do I have to do to assemble a uniformly charged sphere?

To bring in the first dq requires no work since V present is zero.

What do I bring in next (congruous)?

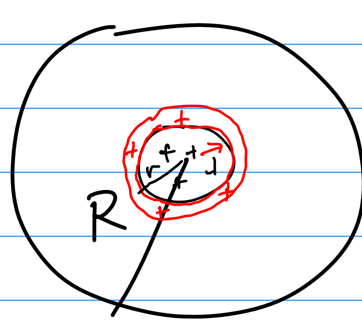
ρ $\frac{\text{Coul}}{m^3}$ is charge density



$$dq = \rho\, 4\pi r^2\, dr$$

$$W = \int V\,dq$$

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{r} \rightarrow \frac{1}{4\pi\epsilon_0}\,\frac{\rho\,\frac{4}{3}\pi r^3}{r}$$



$$W = \int_0^R \frac{1}{4\pi\epsilon_0}\,\rho^2\frac{(4\pi)^2}{3}\,r^4\,dr = \frac{4}{15}\,\frac{\pi\rho^2 R^5}{\epsilon_0}$$

$$= \frac{1}{4\pi\epsilon_0}\,\frac{3}{5}\,\frac{q^2}{R}$$

Questions

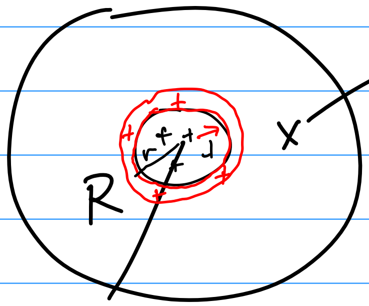
$$W = \int V \, dq$$

Analagous questions about electricity and magnetism?

http://en.wikipedia.org/wiki/Gravitoelectromagnetism

How do I calculate the work if it is easy to determine the voltage at every point in the charge distribution due to all the charges that are present?

This voltage is NOT the voltage due to only the charges that have been brought it!!!



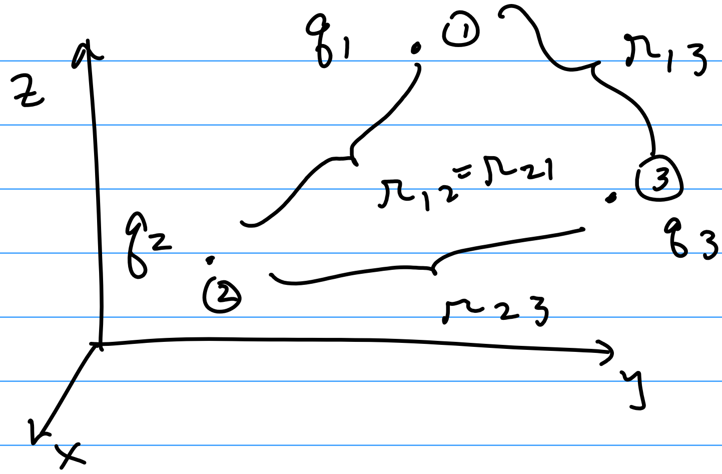
$$V = -\int_{\infty}^{r} \vec{E}_{tot} \cdot d\vec{r}$$

Questions

How do I calculate the work required given the voltage at each point in the charge distribution due to all charges present (congruous)?

What simple exam can I use to understand how this is done (modifying)?

Use a three charges to find work (two is too simple and four too complicated).



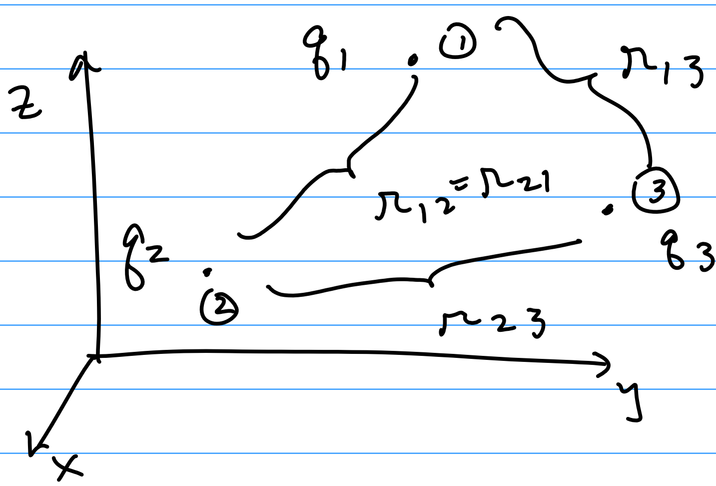
Let charge 1 be brought in first, then charge 2, then charge 3.

$$W_{me} = \frac{1}{4\pi\epsilon_0}\Bigg($$

$$\frac{q_1 q_2}{r_{12}}$$

$$\frac{q_2 q_3}{r_{23}} + \frac{q_1 q_3}{r_{13}}$$

What is the voltage at each charge due to all others (informational)?



$$V_{①} = \frac{1}{4\pi\epsilon_0}\left(\frac{q_2}{r_{12}} + \frac{q_3}{r_{13}}\right)$$

$$V_{②} = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{r_{12}} + \frac{q_3}{r_{23}}\right)$$

$$V_{③} = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{r_{13}} + \frac{q_2}{r_{23}}\right)$$

$$W_{me} = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$

$$2W_{me} = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} + \frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$

$$= \frac{1}{4\pi\epsilon_0}\left[\underbrace{q_1\left(\frac{q_2}{r_{12}} + \frac{q_3}{r_{13}}\right)}_{V(1)} + \underbrace{q_2\left(\frac{q_3}{r_{23}} + \frac{q_1}{r_{12}}\right)}_{V(2)} + \underbrace{q_3\left(\frac{q_1}{r_{13}} + \frac{q_2}{r_{23}}\right)}_{V(3)}\right]$$

$$W_{me} = \frac{1}{2}\sum_{i=1}^{N} q_i V(P_i)$$

↑ i^{th} point

Questions