

Time vs. Frequency domain

Linear response:



$$E_{\text{out}}(w) = E_{\text{in}}(w) e^{\text{reponse}}$$

linear propagation is easiest to describe in freq. space.

- linear systems: $H(w)$ = transfer func., freq. response

$$F_{\text{out}}(w) = H(w) F_{\text{in}}(w)$$

$$\begin{aligned} F_{\text{out}}(t) &= \int \{ H(w) F_{\text{in}}(w) \} \\ &= h(t) \otimes F_{\text{in}}(t) \quad \text{convolution theorem} \\ \text{where} \quad &= \int_{-\infty}^{\infty} h(\tau) f_{\text{in}}(t-\tau) d\tau \quad \text{is convolution} \end{aligned}$$

if input is $F_{\text{in}}(t) = \delta(t)$: impulse

$$F_{\text{out}}(t) = \int_{-\infty}^{\infty} h(\tau) \delta(t-\tau) d\tau = h(t)$$

$$\therefore h(t) = \int_{-\infty}^{\infty} \{ H(w) \} \quad \text{is impulse response.}$$

At a microscopic level, $p(t)$ is the induced dipole when $E(t)$ is, and $P(t) = N_p p(t)$ is the collective response.

When we solve for $\chi''' - N_p P/E$, our method gives $\chi'''(w)$

recall steps: $E(t) = E_0 e^{-i\omega t} + \text{c.c.}$

$$X'''(t) = X_0''' e^{-i\omega t} + \text{c.c.}$$

$$X_0''' = -(\epsilon_{\text{res}}) E_0 / D(w)$$

$$\chi'''(w) = N_p e^2 / m / D(w)$$

So what is the impulse response? For a range of input freq:

$$P''(w) = \mathcal{X}''(w) E(w)$$

Alternative method: take FT of 2nd order eqn.

$$\text{note that } \Im\{\frac{d^2f}{dt^2}\} = -iw F(w)$$

→ eqn for $X''(w)$ with $E(w)$ driving

Now in time domain,

$$P''(t) = \mathcal{F}^{-1}\{\mathcal{X}''(w) E(w)\}$$

Recognize $\mathcal{X}''(w)$ as a transfer function.

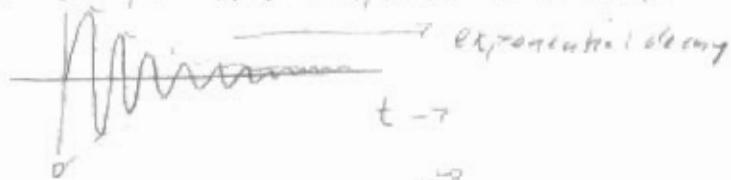
let $R''(t) = 2\pi \mathcal{F}^{-1}\{\mathcal{X}''(w)\}$ ≡ impulse response

Then $P''(t) = \int_{-\infty}^{\infty} R''(\tau) E(t-\tau) d\tau = R'' \otimes E$
(normally have $\frac{1}{2\pi}$ in front, this is absorbed into def'n of R'')

What is the nature of $R''(t)$?

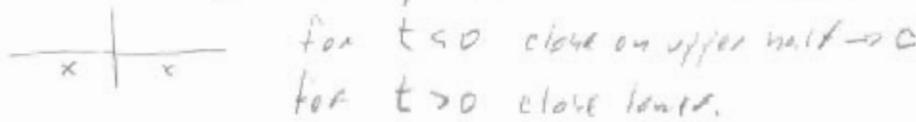
- causality requires $R''(t) = 0$ for $t < 0$

- expect? damped SHD response to a kick:



• Proof. 1) $\mathcal{F}^{-1}\{\mathcal{X}''(w)\} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{iV(\omega/w)}{\omega_0^2 - w^2 - 2\zeta\omega w} e^{-i\omega t} dw$

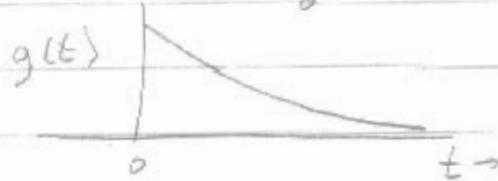
Requires contour integration: poles are off real axis:



for $t < 0$ close on upper half $\rightarrow 0$
for $t > 0$ close lower.

2) work backwards:

$$\mathcal{F}\{g(t)\} = \int_{-\infty}^{\infty} e^{-i\omega t} g(t) dt = \int_0^{\infty} e^{(i\omega - \gamma)t} dt = \frac{-1}{i\omega - \gamma} = G(\omega)$$



$$\mathcal{F}\{g(t) \sin(\omega_R t)\} = \frac{1}{2i} \left(\mathcal{F}\{g(t)e^{i\omega_R t}\} - \mathcal{F}\{g(t)e^{-i\omega_R t}\} \right)$$

$$= \frac{1}{2i} \left(G(\omega + \omega_R) - G(\omega - \omega_R) \right) \quad \text{shift theorem}$$

$$= \frac{1}{2i} \left(\frac{-1}{i(\omega + \omega_R) - \gamma} + \frac{1}{i(\omega - \omega_R) - \gamma} \right)$$

$$= \frac{1}{2} \left(\frac{1}{\omega + \omega_R + i\gamma} + \frac{1}{\omega - \omega_R + i\gamma} \right)$$

$$= \frac{1}{2} \left(\frac{\omega - \omega_R + i\gamma - (\omega + \omega_R + i\gamma)}{\omega^2 - \omega_R^2 - \gamma^2 + 2i\gamma\omega} \right)$$

$$= \frac{\omega_R}{\omega_R^2 - \omega^2 + \gamma^2 - 2i\gamma\omega}$$

compare to dispersion formula:

$$\omega_p^2 = \omega^2 - \gamma^2$$

Time-dependent NL response

generalize 1st order expression:

$$P^{(2)}(t) = \int_0^\infty dt_1 \int_0^\infty dt_2 R^{(2)}(t_1, t_2) E(t-t_1) E(t-t_2)$$

Causality: $R^{(2)} = 0$ for either t_1 or $t_2 < 0$

put $E \rightarrow w$ domain:

$$P^{(2)}(t) = \int_{-\infty}^{\infty} \frac{dw_1}{2\pi} \int_{-\infty}^{\infty} \frac{dw_2}{2\pi} \int_0^\infty dt_1 \left(\int_0^\infty dt_2 R^{(2)}(t_1, t_2) E(w_1) e^{-i w_1 (t-t_1)} E(w_2) e^{-i w_2 (t-t_2)} \right)$$

define $\chi^{(2)}(w_1, w_2; w_1, w_2) = \int_0^\infty dt_1 \int_0^\infty dt_2 R^{(2)}(t_1, t_2) e^{i(w_1 t_1 + w_2 t_2)}$

$$= \sum_{1,2} \{ R^{(2)}(t_1, t_2) \}$$

2D transform
 $R^{(2)}$ truncated $t_1, t_2 \leq 0$

$$\therefore P^{(2)}(t) = \sum_{1,2} \{ \chi^{(2)}(w_1, w_2; w_1, w_2) E(w_1) E(w_2) \}$$

If response is instantaneous $\chi^{(2)}$ is dispersionless.

$$R^{(2)}(t_1, t_2) = \chi^{(2)} \delta(t_1) \delta(t_2)$$

$$P^{(2)}(t) = \chi^{(2)} E(t) E(t)$$

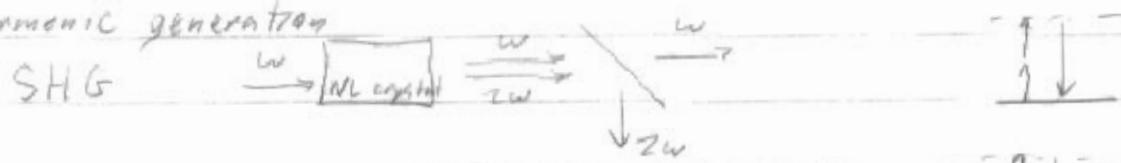
Many systems have delayed response.

Harmonic generation and optical parametric amplifiers.

Motivation:

- Many applications (industrial, commercial, scientific) require specific wavelengths
- cost-effective laser sources are limited
 - Nd:YAG, Ti:sapphire, other solid state.
 - Laser diodes
- gas lasers: HeNe, ion (Ar⁺, Kr⁺), excimer
- dyed lasers
- best to get one laser, convert λ

Harmonic generation



direct THG difficult to phase match.



Nd:YAG λ 's $\lambda_1 = 1064 \text{ nm}$

$$\lambda_2 = 532 \text{ nm}$$

$$\lambda_3 = 355 \text{ nm}$$

$$\lambda_4 = 266 \text{ nm}$$

Tissues.

how to get good efficiency?

- intensity, crystal length, focusing

- phase matching: birefringent / angle tune, quasi-phase matching

Parametric mixing

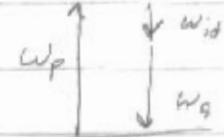
No absorption, instantaneous response.

Sum mixing $\omega_s = \omega_1 + \omega_2$

OPA

$$\omega_{\text{signal}} = \omega_{\text{pump}} - \omega_{\text{idler}}$$

requires a seed, phase matching



Difference-frequency mixing

Same process, no equivalent energy is ω_p, ω_s

can tune across wide λ range, especially with mixing of OPA output.

Example:	pump	800 nm	OPA	SHG
	sig	$1 \mu\text{m} \rightarrow 1.4 \mu\text{m}$		$400 \text{ nm} \rightarrow 700 \text{ nm}$
	idler	$3 \mu\text{m} \rightarrow 1.6 \mu\text{m}$		$1500 \text{ nm} \rightarrow 800 \text{ nm}$

$$\rightarrow -\nabla^2 \vec{E}_n(r) - \frac{\omega_n^2}{c^2} \vec{\epsilon}^{(n)}(\omega_n) \vec{E}_n(r) = \frac{4\pi \omega_n^2}{c^2} \vec{P}_n^{(n)}(r) = \frac{\omega_n^2}{\epsilon_0 c^2} \vec{P}_n^{(n)}(r)$$

What if material is birefringent?

$$\vec{D}_n = \vec{\epsilon}^{(n)}(\omega_n) \cdot \vec{E}_n \quad \text{since } \vec{\epsilon} \text{ is a tensor}$$

∴ change $\vec{\epsilon} \rightarrow \vec{\epsilon}'$ and dot it with \vec{E}

Notes: we have several coupled equations, each at diff ω_n
 - coupling is through $\vec{P}^{(n)}$
 e.g. $\vec{P}^{(2)} = \chi^{(2)} \vec{E}_{\omega_1} \vec{E}_{\omega_2}$
 each of these equations is for 3 vector components.

Application: SFG (laser freq. gen.)

- CW
- 2 inputs at ω_1, ω_2
- outputs at $\omega_3 = \omega_1 + \omega_2$
- plane waves prop in z-direction

$$\vec{E}_n(z, t) = A_n e^{i(k_n z - \omega_n t)} + \text{c.c.}$$

with no nonlinearity, no coupling of separate, linear w.e.

∴ all A_n 's are constant

with nonlinearity, RHS $\rightarrow P^{(n)}$ terms, e.g.

$$P_3^{(2)} = 4\epsilon_0 \text{d}_{322} E_1 E_2 \quad \text{recall } \chi^{(2)} = 2\epsilon_0 \text{d}_{322}$$

$$= 4\epsilon_0 \text{d}_{322} A_1 A_2 e^{i(k_1 z - \omega_1 t + k_2 z - \omega_2 t)}$$

Second -2x from $\omega_1 + \omega_2$ or
 $\omega_2 + \omega_1$

to get this into the NLWE, we have to account

for $A_n(z)$, no dependence on x, y

$$-\frac{d}{dz} \vec{E}_3(z) + \frac{\omega_3^2}{c^2} \vec{E}_3(z) = -\frac{4\omega_3^2}{c^2} \text{d}_{322} \vec{E}_1(z) \vec{E}_2(z)$$