

In order to receive full credit, **SHOW ALL YOUR WORK**. Full credit will be given only if all reasoning and work is provided. Please enclose your final answers in boxes.

1. (10 Points) Conceptual question. Briefly describe the following mathematical concepts.

a. The inner-product of \mathbb{R}^n .

b. The orthogonal complement of a vector space W .

2. (10 Points) Let,

$$\mathbf{y} = \begin{bmatrix} -1 \\ 4 \\ 3 \end{bmatrix}, \quad \mathbf{u}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \quad \mathbf{u}_2 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}.$$

Calculate the distance from \mathbf{y} to the plane $W = \text{span}\{\mathbf{u}_1, \mathbf{u}_2\}$.

3. (10 Points) Let $\mathbf{U}_{n \times n}, \mathbf{V}_{n \times n}$, be orthogonal matrices. Prove that \mathbf{UV} is invertible and also an orthogonal matrix.

4. (10 Points) Given,

$$\mathbf{b}_1 = \begin{bmatrix} 0 \\ 4 \\ 2 \end{bmatrix}, \quad \mathbf{b}_2 = \begin{bmatrix} 5 \\ 6 \\ -7 \end{bmatrix}.$$

Let $B = \{\mathbf{b}_1, \mathbf{b}_2\}$ be a basis for the subspace W of \mathbb{R}^3 . Determine an orthogonal basis for the subspace W .