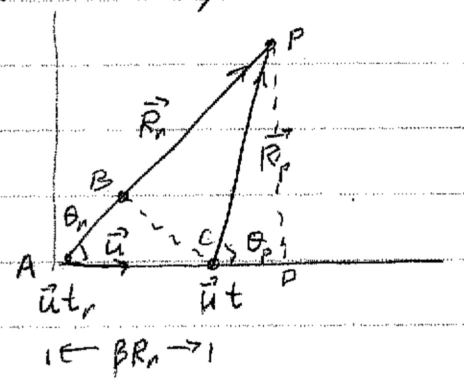
Fields from charge in uniform motion

- no acceleration

$$K = 1 - \frac{\vec{\beta}\cdot\vec{R}_r}{R_r}$$

$$\vec{E} = \left[\frac{e}{K^3 R_r^3}\left(\vec{R}_r - R_r\vec{\beta}\right)\left(1-\beta^2\right)\right]_{t_r}$$

it is possible to write $\vec{E}$ in terms of the present position of the charge:



$t_r = t - R_r/c$

$\vec{u}(t-t_r) + \vec{R}_p = \vec{R}_r$

since $R_r = c(t-t_r)$

$\rightarrow \vec{R}_p = \vec{R}_r - R_r\vec{\beta}$

calc. $[K^2R^2] = K^2R_r^2 = (R_r - \vec{\beta}\cdot\vec{R}_r)^2 = (\overline{AP} - \overline{AB})^2 = \overline{BP}^2$

$\vec{\beta}\cdot\vec{R}_r = \beta R_r \cos\theta_r = \overline{AB}$

put in terms of R_p and θ_p:

$\overline{BP}^2 = R_p^2 - \overline{BC}^2$

$\overline{BC}^2 = \beta^2 R_r^2 - \overline{AB}^2 = \beta^2R_r^2 - \beta^2R_r^2\cos^2\theta_r = \beta^2R_r^2\sin^2\theta_r$

Now $R_r\sin\theta_r = \overline{DP} = R_p\sin\theta_p$

$\therefore K^2R_r^2 = R_p^2 - \beta^2R_p^2\sin^2\theta_p = R_p^2(1-\beta^2\sin^2\theta_p)$

all together: $$\vec{E} = \frac{e\vec{R}_p(1-\beta^2)^*}{R_p^3(1-\beta^2\sin^2\theta_p)^{3/2}}$$

$\vec{B} = \frac{\vec{R}\times\vec{E}}{R}$

$= \left(\frac{\vec{R}_p}{R_r} + \vec{\beta}\right)\times\vec{E} \rightarrow \vec{B} = \vec{\beta}\times\vec{E} \rightarrow$ same w/ $\vec{\beta}\times\vec{R}_p$

Radiation from accelerated charges $\rightarrow$ $\boxed{\text{Larmor } P_{rad} = \frac{2}{3}\frac{e^2 a^2}{c^3}}$

- at far distance from source, only radiation terms matter.

$$\vec{E}_a = \frac{e}{c^2 K^3 R^3} \vec{R} \times \left((\vec{R} - \vec{\beta} R) \times \vec{a} \right) \Big|_{t_r}$$

for most part, we're interested in calculating radiated power

- leave it understood that the emitted power is evaluated at t_r.
- suppress [ ] notation.

Even at low velocities, $\beta \ll 1$, accel $\rightarrow$ radiation.

let $\beta \rightarrow 0$ $K \rightarrow 1$

$$\vec{E}_a = \frac{e}{c^2 R^3} \vec{R} \times (\vec{R} \times \vec{a}) = \frac{e}{c^2 R^3} \left(\vec{R}(\vec{R} \cdot \vec{a}) - \vec{a} R^2 \right)$$

$B(A \cdot C) - C(A \cdot$

note $\vec{E}_a \perp \vec{R}$ transverse

$$\vec{B}_a = \frac{\vec{R} \times \vec{E}_a}{R}$$

$$\vec{S}_a = \frac{c}{4\pi} \vec{E}_a \times \vec{B}_a = \frac{c}{4\pi} \vec{E}_a \times \left(\frac{\vec{R} \times \vec{E}_a}{R} \right) = \frac{c}{4\pi R} \left(\vec{R} E_a^2 - \vec{E}_a \underbrace{\cancel{(\vec{E}_a \cdot \vec{R})}}_{0} \right)$$

$\vec{E}_a \perp \vec{R}$

$$\therefore \vec{S}_a = \frac{c}{4\pi} E_a^2 \hat{R}$$

in direction outward from retarded position

evaluate:

$$E_a^2 = \frac{e^2}{c^4 R^6} \left(\vec{R}(\vec{R} \cdot \vec{a}) - \vec{a} R^2 \right)^2 = \frac{e^2}{c^4 R^6} \left(R^2 (\vec{R} \cdot \vec{a})^2 + a^2 R^4 - 2R^2 (\vec{R} \cdot \vec{a})^2 \right)$$

$$= \frac{e^2}{c^4 R^4} \left(R^2 a^2 - (\vec{R} \cdot \vec{a})^2 \right) = \frac{e^2 a^2}{c^4 R^2} \underbrace{(1 - \cos^2\theta)}_{\sin^2\theta}$$

$$\vec{S}_a = \frac{c}{4\pi} \frac{e^2 a^2 \sin^2\theta}{c^4 R^2} \frac{\vec{R}}{R} = \underline{\frac{e^2 a^2}{4\pi c^3 R^2} \sin^2\theta \, \hat{R}}$$

Express in terms of the angular distribution.

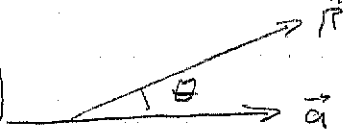
total power $= \quad P = \oint \vec{S}_a \cdot d\vec{A} = \oint \underbrace{\vec{S}_a \cdot \hat{R}}_{\equiv dP/d\Omega} \underbrace{R^2 \sin\theta\, d\theta\, d\phi}_{d\Omega}$

$$\frac{dP}{d\Omega} = \frac{e^2 a^2}{4\pi c^3} \sin^2\theta$$

note: this is evaluated at the particle's local time

θ is measured from $\vec{a}$

$dP/d\theta \to 0$ at $\theta = 0$ (forward)



$\to$ max at $\theta = \pi/2$ transverse.

total rad. power:

$$P = \int_{4\pi} \left(\frac{dP}{d\Omega}\right) d\Omega = 2\pi \int_0^\pi \frac{e^2 a^2}{4\pi c^3} \sin^2\theta \sin\theta\, d\theta$$

$$= \frac{e^2 a^2}{2c^3} \underbrace{\int_0^\pi \sin^3\theta\, d\theta}_{= 4/3}$$

$$\boxed{P = \frac{2}{3}\frac{e^2 a^2}{c^3}}$$

Larmor radiation formula.

remember this!