Propagation of non monochromatic light.

single frequency:

$$E(t) = E_0 e^{-i\omega_0 t}$$

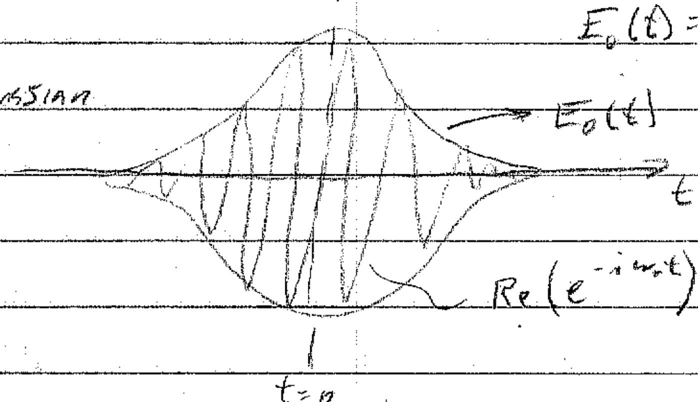
quasimonochromatic:

$$E(t) = E_0(t) e^{-i\omega_0 t}$$

ω_0 = carrier freq.

$E_0(t)$ = envelope.

ex. Gaussian



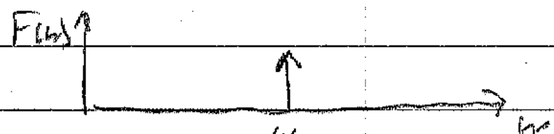
Any modulation in the time domain requires a mixture of frequencies. Use Fourier transforms to get spectrum:

$$\text{Define} \quad \mathcal{F}\{f(t)\} \equiv F(\omega) = \int_{-\infty}^{\infty} f(t) e^{i\omega t} dt$$

$$\mathcal{F}^{-1}\{F(\omega)\} = f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega) e^{-i\omega t} d\omega$$

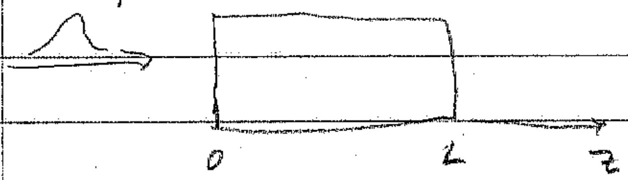
- note opposite signs in $\mathcal{F}\{\ \}$ and $\mathcal{F}^{-1}\{\ \}$
- choice in signs is consistent with $e^{-i\omega_0 t}$ convention

$$\mathcal{F}\{e^{-i\omega_0 t}\} = \int_{-\infty}^{\infty} e^{i(\omega-\omega_0)t} dt \equiv 2\pi\delta(\omega-\omega_0)$$



- note placement of 2π factor.

Dispersion



by travelling from $z=0$ to $z=L$,

$$E(z=0)\,e^{ik_0 nL}$$

since $n = n(\omega)$ we must apply this in the freq. domain

$$\tilde{E}_{out}(\omega) = \tilde{E}_{in}(\omega)\,e^{i\frac{\omega}{c}n(\omega)L}$$

note that if $n(\omega)$ is real,

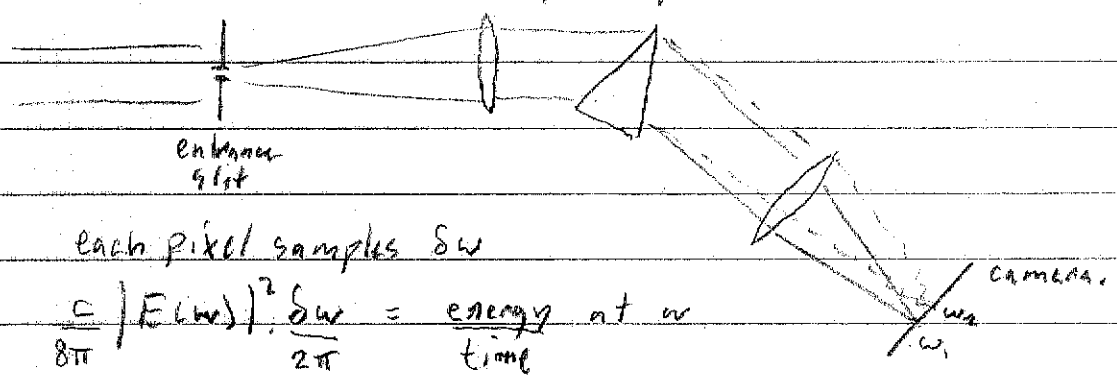
$|E_{out}(\omega)|^2$ is unchanged by medium

$\frac{\omega}{c}n(\omega)L$ is just a phase shift, depends on ω.

$\equiv \phi(\omega)$ spectral phase.

$|E_{out}(\omega)|^2 \propto$ spectral intensity

this is what is measured by a spectrometer:



each pixel samples $\delta\omega$

$$\frac{c}{8\pi}|E(\omega)|^2\frac{\delta\omega}{2\pi} = \frac{\text{energy}}{\text{time}} \text{ at } \omega$$

Gaussian pulse propagating through a dispersive medium

let $E_{in}(t) = E_0 a(t) e^{-i\omega_0 t}$

with $a(t) = \exp(-t^2/\tau^2)$

here no extra phases

calc. spectrum: $A(\omega) = \mathcal{F}\{a(t) e^{-i\omega_0 t}\}$

1) $\mathcal{F}\{e^{-t^2/\tau^2}\} = \int_{-\infty}^{\infty} e^{-t^2/\tau^2 + i\omega t}\, dt$

complete square in exponent to get to form:

$$\int_{-\infty}^{\infty} e^{-u^2} du = \sqrt{\pi}$$

$$-\frac{1}{\tau^2}(t^2 - i\omega\tau^2 t) = -\frac{1}{\tau^2}\left[\left(t - \frac{i\omega\tau^2}{2}\right)^2 + \frac{\omega^2\tau^4}{4}\right]$$

$$\rightarrow e^{-\frac{\omega^2\tau^2}{4}} \int_{-\infty}^{\infty} e^{-\frac{(\ )}{\tau^2}} dt$$

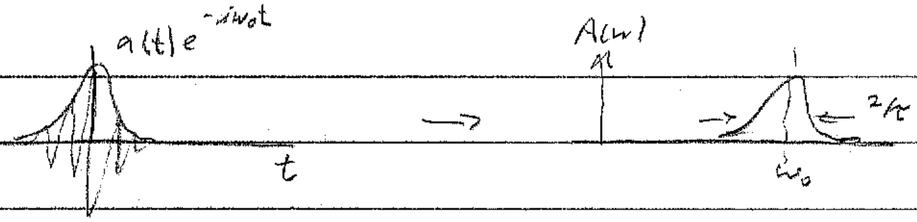
let $u = \left(t - \frac{i\omega\tau}{2}\right)/\tau \qquad du = \frac{1}{\tau} dt$

$$\mathcal{F}\{e^{-t^2/\tau^2}\} = \tau \cdot \sqrt{\pi}\, e^{-\omega^2\tau^2/4}$$ another Gaussian.

2) use shift theorem:

$$\int f(t) e^{-i\omega_0 t} e^{i\omega t} dt = F(\omega - \omega_0)$$

$$A(\omega) = \sqrt{\pi\tau^2}\, e^{-(\omega-\omega_0)^2\tau^2/4}$$



in t: $1/e$ half width $= \tau$

in ω: " $= 2/\tau$

$\Delta t \, \Delta\omega = 2$ uncertainty principle

QM: photon energy $= \hbar\omega$

$\Delta E \, \Delta t = 2\hbar$ note QM definition of width is different.

photon counting: $\hbar\omega$ can be anywhere w/in range $\Delta\omega$

individual photons share same wavefunction as packet

$E \sim \psi$

$|E|^2 \sim |\psi|^2$ energy density ~ probability density

$E(t)$ and $\tilde{E}(\omega)$ are different <u>representations</u> of same signal

∴ expect

$$\int |E(t)|^2 \, dt = \frac{1}{2\pi} \int |E(\omega)|^2 \, d\omega$$

↳ comes in from proof

this is Parseval's thm.

check:

$$\int E_0^2 e^{-2t^2/\tau^2} \, dt = E_0^2 \frac{\tau}{\sqrt{2}} \int e^{-u^2} \, du = E_0^2 \tau \sqrt{\frac{\pi}{2}}$$

$$\frac{1}{2\pi} \int E_0^2 \pi \tau^2 e^{-(\omega-\omega_0)^2 \tau^2/2} \, d\omega = \frac{\tau^2}{2} E_0^2 \sqrt{\frac{2}{\tau^2}} \sqrt{\pi} \quad ✓$$

$\frac{c}{4\pi} |E(t)|^2$ = intensity (time dependent) $= I(t)$ e.g. W/cm²

$\int I(t) \, dt$ = energy fluence e.g. J/cm²