Practice Midterm Exam Problems for PH311

version 1.1

fixed a typo on the second problem on page 3

We reserve the right to add problems and fix typos. This is version 1.0. Remember, the real exam will have only 3 problems that you must do. So there are lots of extra problems for you to play with in this set.

- In this problem we consider a diagonal 2×2 matrix:

$$\mathbf{A} = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} . \tag{1}$$

  In this expression λ_1 and λ_2 are constants.

  Problem a: Show that

$$\mathbf{A}^n = \begin{pmatrix} \lambda_1^n & 0 \\ 0 & \lambda_2^n \end{pmatrix} . \tag{2}$$

  Problem b: Derive that

$$\ln(\mathbf{I} + \mathbf{A}) = \begin{pmatrix} \ln(1+\lambda_1) & 0 \\ 0 & \ln(1+\lambda_2) \end{pmatrix} . \tag{3}$$

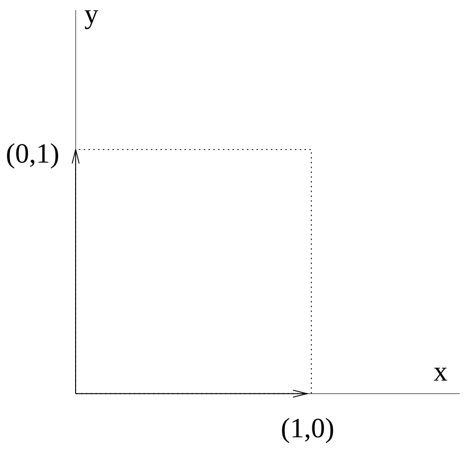
- Use the McClaurin series expansion of $\cos(x)$ to thow that

$$\lim_{x \to \infty} \frac{1 - \cos(x)}{x^2} = 1/2$$

- Figure out the column space and null space of the following two matrices:

$$\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} \text{ and } \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{4}$$

- Here is a box generated by two unit vectors, one in the x direction and one in the y direction.



If we take a two by two matrix

$$A = \begin{bmatrix} a & b \\ b & d \end{bmatrix}$$

and apply it to the two unit vectors, we get two new vectors that form a different box. (I.e., take the dot product of A with the two column vectors $(1,0)^T$ and $(0,1)^T$.) Draw the resulting boxes for the following matrices and say in words what the transformation is doing.

1. $$\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

2. $$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

3. $$\begin{bmatrix} 2 & 0 \\ 0 & 1/2 \end{bmatrix}$$

4. $$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

5. $$\begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}$$

- Consider the following linear system.

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Show that the right-hand side is not in the column space of the matrix, and hence no ordinary solution exists.

- Use your knowledge of geometric series to derive an exact expression for the following sum:

$$\sum_{k=m}^{n} x^k$$

for all $m \leq n$ and all $x \neq 1$.

- Show that

$$\sum_{k=0}^{\infty} kx^k = \frac{x}{(1-x)^2}$$

- Suppose

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}$$

Show that if $\alpha = 1$ and $\beta = \gamma = 0$, then the right-hand side is in the column space of the matrix. What is the solution in this case. Now suppose that $\alpha = \beta = 0$ and $\gamma = 1$. Show that the right-hand side is not in the column space.

- Let

$$A = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$C = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Fill in the following multiplication table. In other words the the box in the second row and second column should contain A^2. What you should see is that the table can be completely filled with the four matrices A, B, C, I and hence these four matrices are closed under multiplication. Since they have an identitiy, this makes them a group.

	A	B	C	I
A				
B				
C				
I				

- Let x and y be vectors and T a matrix. Show by giving a counterexample that for a given x, y, T is not uniquely specificied by

$$y = Tx$$

. This shoes that division by vectors is not in general well-defined.