MACS 332A - Fall 2008

NAME:__________________________________ **Exam I**

In order to receive full credit, SHOW ALL YOUR WORK. Full credit will be given only if all reasoning and work is provided. Please enclose your final answers in boxes.

1. Given the system of equations,

$$\begin{array}{rcrcrcr} & & x_2 & + & 3x_3 & = & 8, \\ 2x_1 & & & - & 4x_3 & = & -10, \\ -2x_1 & - & 2x_2 & - & 2x_3 & = & -6 \\ 3x_1 & + & 2x_2 & & & = & 1 \end{array}$$

   (a) (12 points) Solve the system by using row operations to get an augmented matrix in reduced row-echelon form. Write the solution in parametric-vector form.

   (b) (5 points) Find the solution for the homogeneous system corresponding to the given system. Describe the geometric relationship between the homogeneous solution and the nonhomogeneous solution found in part (a)

   (c) (5 points) Do the columns of the coefficient matrix for the system span $\mathbb{R}^4$? Justify your answer.

2. (7 points) Suppose that the coefficient matrix corresponding to a linear system is a 5×7 matrix and has 4 pivot columns. How many pivot columns does the augmented matrix have if the linear system is inconsistent?

3. Construct a set of four vectors in $\mathbb{R}^4$ that are

   (a) (6 points) linearly independent

   (b) (6 points) linearly dependent

4. (12 points) Let $A = \begin{bmatrix} -3 & 1 & 4 \\ 6 & 0 & -2 \\ -6 & 2 & 8 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 3 \\ -8 \\ 6 \end{bmatrix}$. Solve the equation $A\mathbf{x} = \mathbf{b}$ using an LU factorization.

5. (10 points) Let A and B be invertible $n \times n$ matrices. Show that $(A^{-1}(B^{-1})^T)^T$ is the inverse of $A^T B$.

6. (12 points) Let $A = \begin{bmatrix} 0 & 5 & 0 & 2 & 0 \\ 0 & 2 & 0 & -3 & -1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -3 & 0 & 0 \\ 5 & 5 & 0 & 0 & 0 \end{bmatrix}$. Find $\det A$.

7. (11 points) Let A be a 4×5 matrix, let $\mathbf{y}_1$ and $\mathbf{y}_2$ be vectors in $\mathbb{R}^4$, and let $\mathbf{w} = \mathbf{y}_1 + \mathbf{y}_2$. Suppose $\mathbf{y}_1 = A\mathbf{x}_1$ and $\mathbf{y}_2 = A\mathbf{x}_2$ for some vectors $\mathbf{x}_1$ and $\mathbf{x}_2$ in $\mathbb{R}^5$. Explain why the system $A\mathbf{x} = \mathbf{w}$ is consistent.

8. (12 points) Given the system of equations

$$\begin{aligned} x_1 + 3x_2 &= k \\ -2x_1 + hx_2 &= 8 \end{aligned}$$

(a) find all values of h and k fo which the system has a unique solution.

(b) find all values of h and k for which the system has infinitely many solutions.