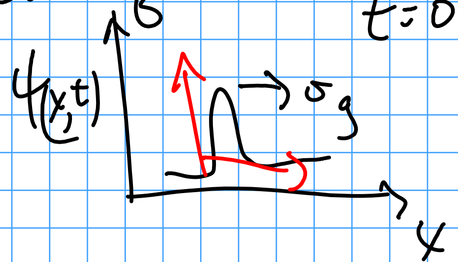
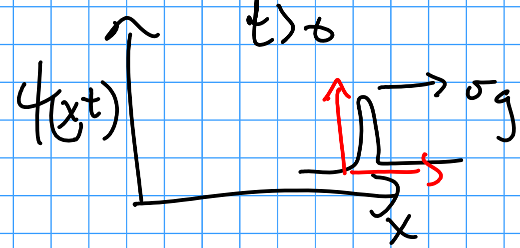
Non-linear PDE ($\vec{\nabla}\cdot\vec{J} = -\frac{\partial \rho}{\partial t}$) is often solved by

METHOD OF CHARACTERISTICS (look at wikipedia)

- Casts the PDE into an ODE via a transformation

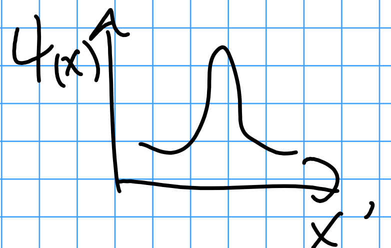
Wave eqn





$$\frac{\partial^2 \psi(x,t)}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 \psi}{\partial t^2}$$

Go to frame of pulse $x' = x + v_g t$



$$\frac{d^2 \psi(x)}{dx^2} = f(x)$$

$$\vec{\nabla}\times\vec{B} = \mu_0 \vec{J} \qquad \vec{B} = \vec{\nabla}\times\vec{A}$$

$$\vec{\nabla}\times\vec{\nabla}\times\vec{A} = \mu_0 \vec{J}$$

Given $\vec{A}$ find $\vec{J}$

Given $\vec{J}$ have a PDE in $\vec{A}$

$$\parallel$$

$$\vec{\nabla}(\underbrace{\vec{\nabla}\cdot\vec{A}}_{=0}) - \nabla^2 \vec{A} = \mu_0 \vec{J}$$

Lorentz gauge

$\nabla^2 \vec{A} = -\mu_0 \vec{J}$ Look up DEL in cylindrical coordinates on wikipedia for $\nabla^2 \vec{A}$

$\nabla^2 A_x = -\mu_0 J_x$ $\quad$ $\nabla^2 A_y = -\mu_0 J_y$ $\quad$ $\nabla^2 A_z = -\mu_0 J_z$

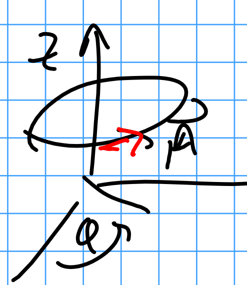
$\nabla^2 V = -\rho/\epsilon_0$ $\quad$ given ρ $\quad$ $V = \int \frac{k\,dq}{r}$ $\quad$ $dq = \rho\, d\tau'$

$A_x = \frac{\mu_0}{4\pi}\int \frac{J_x\, d\tau'}{r}$ $\quad$ given J find $\vec{A}$

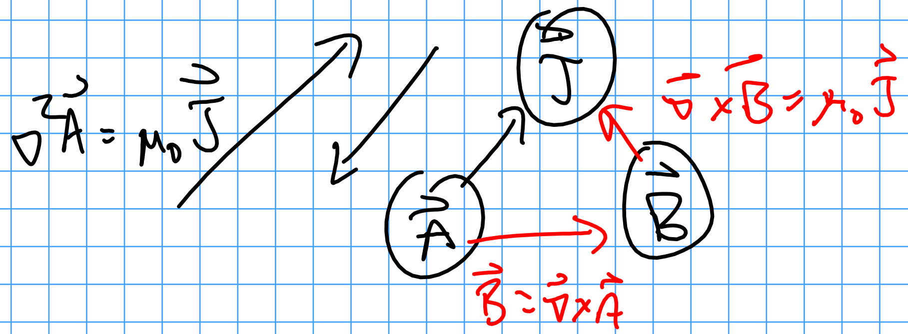
$$\begin{tikzcd}[column sep=huge, row sep=huge]
 & \vec{J} \arrow[dl, "{\vec{A} = \frac{\mu_0}{4\pi}\int \frac{\vec{J}\, d\tau'}{r}}"'] \arrow[dr, "{\vec{B} = \frac{\mu_0}{4\pi}\int \frac{\vec{J}\times\hat{r}\, d\tau'}{r^2}}"] & \\
\vec{A} \arrow[rr, "\vec{B} = \vec{\nabla}\times\vec{A}"] & & \vec{B}
\end{tikzcd}$$

Ex: What J produces A_φ = const $\quad A_s = 0 \quad A_z = 0$

Cylindrical coords



We are given $\vec{A}$ want find $\vec{J}$



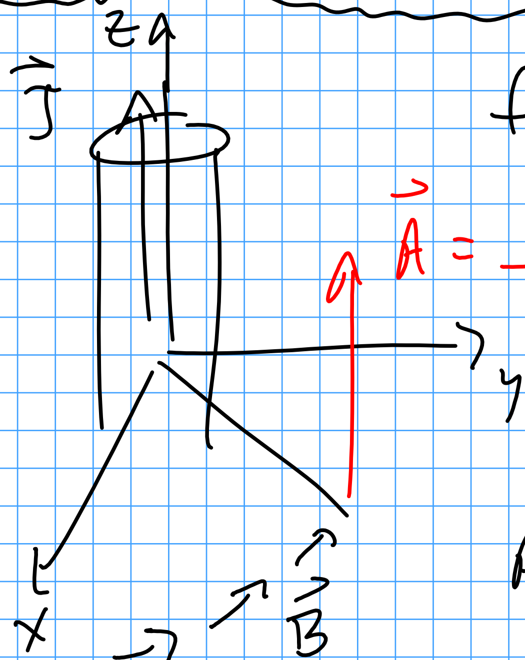
method 1.) $\nabla^2 \vec{A}$ look up in cylindrical coords

& solve for $\mu_0 \vec{J}$

method 2.) $\vec{B} = \vec{\nabla} \times \vec{A} = \frac{1}{s} \frac{\partial}{\partial s} (s \underbrace{k}_{A_\phi}) \hat{z} = \frac{k}{s} \hat{z}$

$$\vec{\nabla} \times \vec{B} = + \frac{\partial}{\partial s} \left(\frac{k}{s} \hat{\phi} \right) = \qquad \vec{J} = -\frac{1}{\mu_0} \left(\frac{\partial}{\partial s} \left(\frac{k}{s} \right) \right) \hat{\phi}$$

Ex! $\vec{J}$



find $\underline{\underline{\vec{A}}}$ given $\underline{\underline{\vec{J}}}$

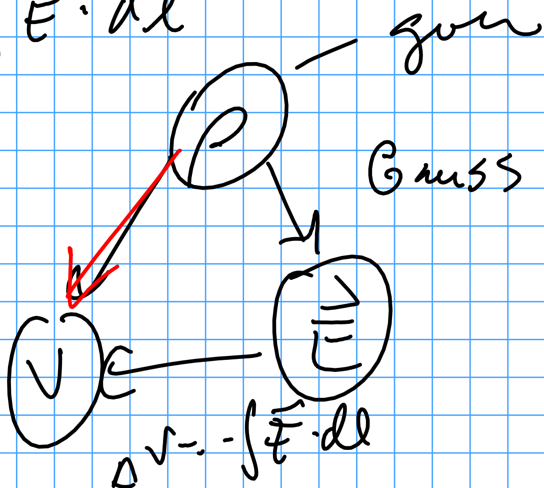
$\vec{A} = \_\_\ \hat{z}$

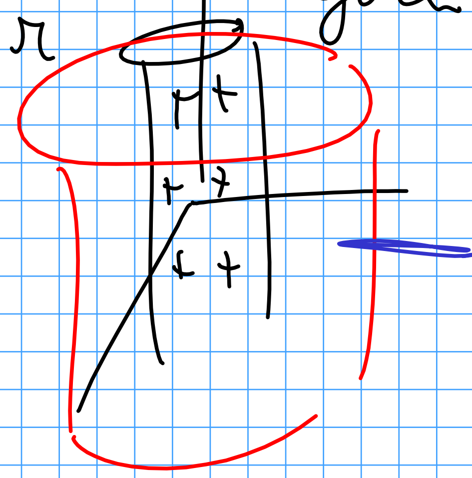
$$A_z = \frac{\mu_0}{4\pi} \int \frac{J_z \, d\tau'}{|\vec{r} - \vec{r}\,'|}$$

method 1.) analogy with electrostatics

$$V \propto \int \frac{k\,dq}{r} = \frac{1}{4\pi\epsilon_0}\int \frac{\rho\, d\tau'}{r}$$ given ρ

$$\Delta V = -\int \vec{E}\cdot d\vec{l}$$

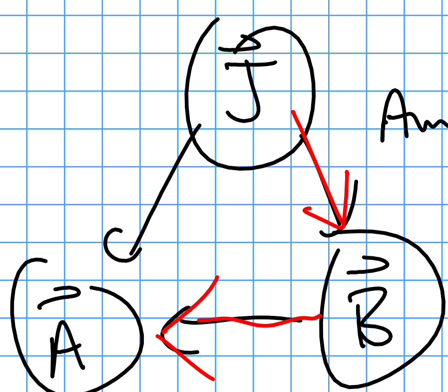




$$E \propto \frac{1}{r}$$

$$\int \vec{E}\cdot d\vec{r} \propto \int \frac{dr}{r}$$

Method 2.) know that $\vec{A} = A_z \hat{z}$



Amps law gives $\vec{B} = \frac{\mu_0}{2\pi}\frac{I_0}{r}\hat{\phi}$

$$\nabla\times\vec{A} = \vec{B} = \frac{\mu_0}{2\pi}\frac{I}{s}\hat{\phi}$$

$s \leftrightarrow r$

$$\nabla\times\vec{A} = \frac{\partial A}{\partial s}\hat{\phi} = \frac{\mu_0}{2\pi}\frac{I_0}{s}\hat{\phi}$$

$A_z\hat{z}$

know $\vec{A}$ is in $\hat{z}$ direction

$$\frac{dA}{ds} = \frac{\mu_0 I_0}{2\pi s}$$