distance s are measured from the x axis. The distance s is related to the polar coordinates r and θ by

$$\theta = \frac{s}{r} \qquad s = r\theta \tag{3-39}$$

Since r is constant, the speed of the particle is

$$v = \frac{ds}{dt} = r\frac{d\theta}{dt} \tag{3-40}$$

We define $\hat{\mathbf{r}}$ to be a unit vector parallel to the position vector $\mathbf{r}$ and $\hat{\boldsymbol{\theta}}$ to be a unit vector perpendicular to $\mathbf{r}$, tangent to the circle in the direction of increasing θ and s.

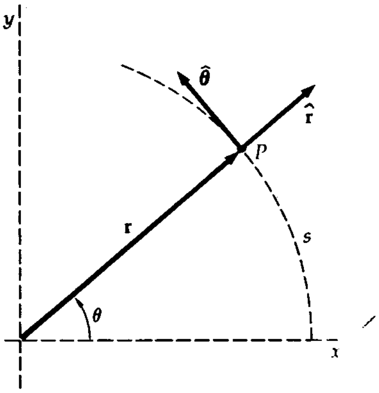

Figure 3-27
Definition of the unit vectors $\hat{\mathbf{r}}$ and $\hat{\boldsymbol{\theta}}$.

The position, velocity, and acceleration vectors for a particle moving in a circle of radius r are

$$\mathbf{r} = r\hat{\mathbf{r}} \tag{3-41}$$

$$\mathbf{v} = v\hat{\boldsymbol{\theta}} = \frac{ds}{dt}\hat{\boldsymbol{\theta}} = \frac{r\,d\theta}{dt}\hat{\boldsymbol{\theta}} \tag{3-42}$$

$$\mathbf{a} = -\frac{v^2}{r}\hat{\mathbf{r}} + \frac{dv}{dt}\hat{\boldsymbol{\theta}} = -\frac{(ds/dt)^2}{r}\hat{\mathbf{r}} + \frac{d^2s}{dt^2}\hat{\boldsymbol{\theta}} \tag{3-43}$$

Unit vectors $\hat{\mathbf{r}}$ and $\hat{\boldsymbol{\theta}}$

We can relate the unit vectors $\hat{\mathbf{r}}$ and $\hat{\boldsymbol{\theta}}$ to the rectangular unit vectors $\mathbf{i}$ and $\mathbf{j}$. From Figure 3-28 we have

$$\mathbf{r} = r\cos\theta\,\mathbf{i} + r\sin\theta\,\mathbf{j} \tag{3-44}$$

Therefore

$$\hat{\mathbf{r}} = \frac{\mathbf{r}}{r} = \cos\theta\,\mathbf{i} + \sin\theta\,\mathbf{j} \tag{3-45}$$

Similarly from Figure 3-28 we see that $\hat{\boldsymbol{\theta}}$ has rectangular components $\hat{\theta}_x = -\sin\theta$ and $\hat{\theta}_y = \cos\theta$; thus

$$\hat{\boldsymbol{\theta}} = -\sin\theta\,\mathbf{i} + \cos\theta\,\mathbf{j} \tag{3-46}$$

The unit vectors $\hat{\mathbf{r}}$ and $\hat{\boldsymbol{\theta}}$ differ from $\mathbf{i}$ and $\mathbf{j}$ in that their direction depends on r and θ, that is, on the location of the particle. Thus the unit vectors $\hat{\mathbf{r}}$ and $\hat{\boldsymbol{\theta}}$ change with time as the particle moves in a circle, while the unit vectors $\mathbf{i}$ and $\mathbf{j}$ are true constant vectors.

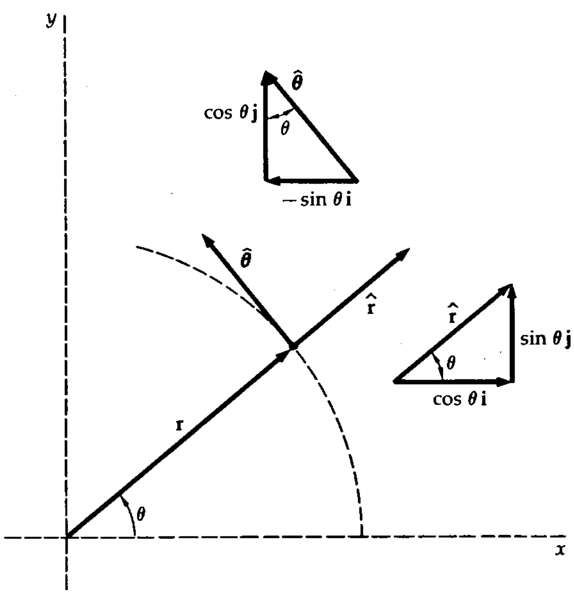

Figure 3-28
The unit vectors $\hat{\mathbf{r}}$ and $\hat{\boldsymbol{\theta}}$ are related to $\mathbf{i}$ and $\mathbf{j}$ by $\hat{\mathbf{r}} = \cos\theta\,\mathbf{i} + \sin\theta\,\mathbf{j}$ and $\hat{\boldsymbol{\theta}} = -\sin\theta\,\mathbf{i} + \cos\theta\,\mathbf{j}$.