Note Title 1/29/2010

Exam I Monday Feb 1

I expect you to

⇒ be able to apply Gauss's Law:
- choose appropriate Gaussian surface
- calculate flux
- calculate $Q_{enclosed}$
- put these together to find E
- know how to apply the differential form of Gauss's Law
- check answer

⇒ be able to find $\vec{E}$ from a continuous charge distribution
- calculate dq in various coordinate systems
- calculate $\vec{r}$
- determine appropriate limits on the integral

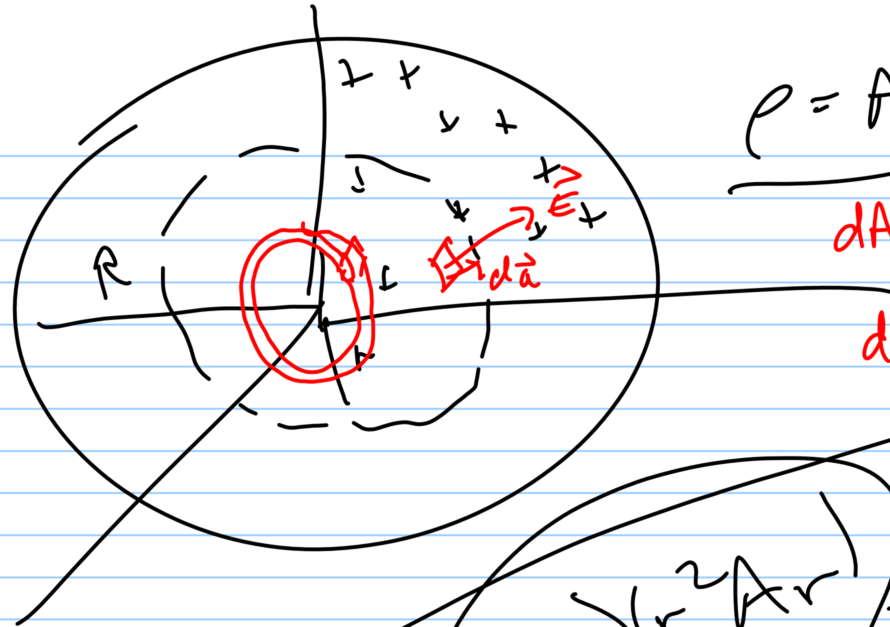
- check your answer

⇒ - be able to determine $\int \vec{E} \cdot d\vec{l}$ along simple paths in various coordinate systems

- be able to determine $\oint \vec{E} \cdot d\vec{a}$ for simple surfaces in various coordinate systems

- no delta functions
- no bndy condition

Ex: Find $\vec{E}$ inside



$$\rho = Ar$$

$$dA = 4\pi r^2$$

$$d\tau = 4\pi r^2 dr$$

$$\vec{\nabla}\cdot\vec{A} = \frac{1}{r^2}\frac{\partial(r^2 A_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(A_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\varphi}{\partial\varphi}$$

$$\vec{A} = A_r\hat{r} + A_\theta\hat{\theta} + A_\varphi\hat{\varphi}$$

$$Q_{enc} = \int \rho\, d\tau = \int_0^{r} A r' 4\pi r'^2 dr' = 4\pi \frac{A r^4}{4}$$

same on all tiles

$$\oint \vec{E}\cdot da = \oint |\vec{E}|\,|d\vec{a}|\cos\phi = E\underbrace{\oint |d\vec{a}|}_{\text{area sphere}} = E\,4\pi r^2$$

all tiles

$$E\,4\cancel{\pi} r^2 = \frac{\cancel{\pi} A r^4}{\epsilon_0}$$

$$\vec{E} = \frac{A r^2}{4\epsilon_0}\hat{r} = A_r\hat{r} + \phi\hat{\theta} + \phi\hat{\varphi}$$

$$\oint \vec{E}\cdot d\vec{a} = \frac{Q_{in}}{\epsilon_0} = \int \frac{\rho\, d\tau}{\epsilon_0}$$

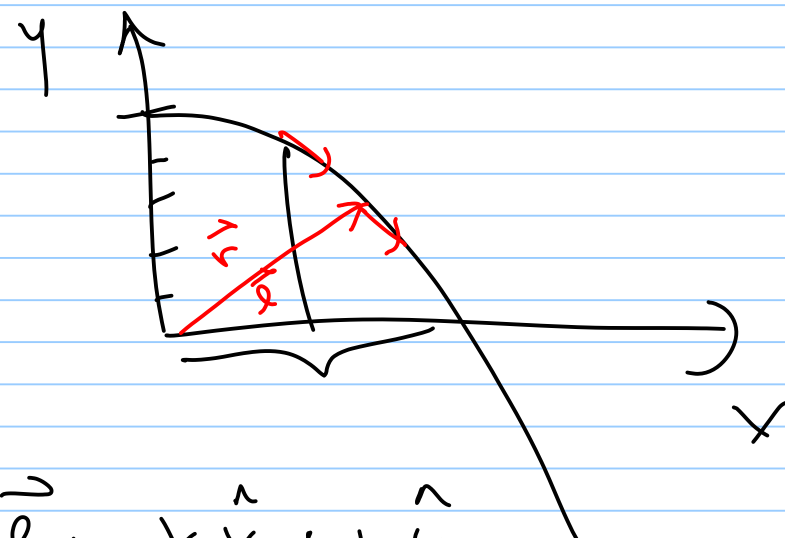
$\downarrow$ div th

$$\int \underline{\vec{\nabla}}\cdot\vec{E}\,d\tau = \int \frac{\rho\,d\tau}{\epsilon_0}$$

$$\vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0$$

$$\vec{\nabla}\cdot\vec{E} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{A r^2}{4\epsilon_0}\right) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^4 \frac{A}{4\epsilon_0}\right) = \frac{1}{\cancel{r^2}}\cancel{4}r^{\cancel{3}}\frac{A}{\cancel{4}\epsilon_0} = \frac{Ar}{\epsilon_0}$$

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0} = \frac{Ar}{\epsilon_0} \qquad \rho = Ar$$



$$y = 5 - 4x^2$$

$$d\vec{l} = ?$$

$$\vec{r} = \vec{l} = x\hat{x} + y\hat{y}$$
$$= x\hat{x} + (5-4x^2)\hat{y} \Rightarrow d\vec{l} = dx\,\hat{x} - 8x\,dx\,\hat{y}$$

$\int |d\vec{l}|$ give distance tra.

$$\int \vec{E} \cdot d\vec{l} = \int (E_x \hat{x} + E_y \hat{y}) \cdot (dx \hat{x} - 8x\,dx\,\hat{y})$$

$$= \int_0^{\sqrt{\frac{5}{4}}} E_x\,dx - \int_0^{\sqrt{\frac{5}{4}}} E_y\,8x\,dx$$

$5 - 4x^2 = 0 \qquad x_f = \sqrt{\frac{5}{4}}$