

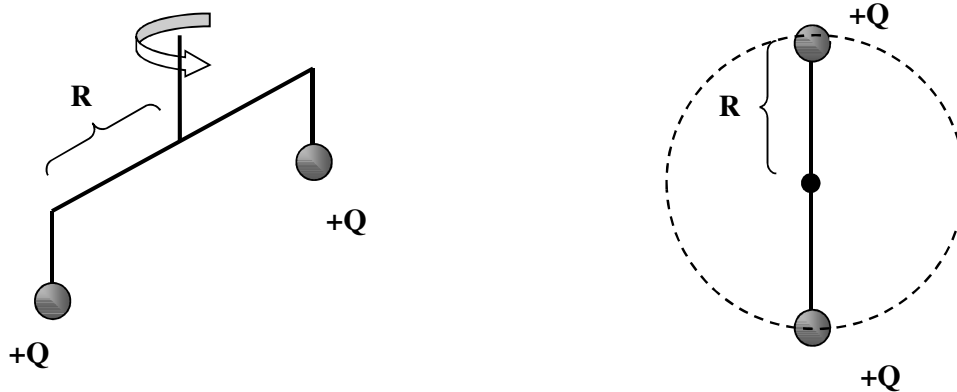
PHGN 462 Homework 1

- 1) a) Write down the Maxwell equations in differential form. Then describe, in words and sentences, what each of them is saying. And by that I mean talk about the physical content – your answer should not be of the form “This one is saying that $\text{div } \mathbf{E} = \rho / \epsilon_0$.”
- b) Show me how to get from the differential form of Faraday’s law to the integral form. If you’re restricting the situation somehow, make sure you explain how and why. Then show me how to get from the integral form to the differential form. Explain all the steps.
- c) Show me how the Maxwell equations need to change if we’re to represent magnetic monopoles. Explain the changes. This information is pretty common (I’ve got a youtube video covering it myself, even), and you should feel free to look it up, but take the time to understand what you’re reading and to describe it in your own words.

Note that in recent years people have observed collective magnetic effects that have monopole-like features. However, the real prize, true monopoles that exist as discrete particles, are still nowhere to be found.

- 2) (note: This problem is intended to be review. Don’t try to do separation of variables or anything like that. Just use Faraday’s law to find the induced E-field and go from there)

The picture shows two balls hanging from a crossbar that’s free to rotate (like in a mobile, if anyone knows what those are anymore). Each ball has some positive charge Q and mass M . The crossbar is of total length $2R$. There’s a perspective view (left) and a top-down view (right). The arrow shows one possible rotation; it’s free to rotate in either direction.



Now let’s suppose the region inside the dotted circle is filled with a uniform magnetic field \mathbf{B} oriented into the page. That field is turned off linearly over some time interval of length T (that is, B as a function of time is a linear function, and takes T to go from max value to zero). Describe the ensuing motion of the balls, and calculate their final angular momentum

Since we started with no motion and ended with motion, and changed only the fields, we kind of have to conclude that fields, even *static* ones, can have both linear and angular momentum if we want those quantities to be conserved (and just to be clear, we want that very much). We'll be talking about field momentum in more detail soon enough.

3) (like problem 2, this is review and shouldn't require any terribly exotic methods)

a) Take a nice, long solenoid of radius a and coil density n . It's long enough that it's a pretty good approximation to call it infinite. If there's some steady current I_0 running through it, what is the B-field inside and outside the solenoid? You don't have to derive it – just write it down.

b) Now suppose that that current is varying in time. In particular $I(t) = I_0 + kt$ for some positive constant t . Derive an expression for the E-field (if any) present in the system, both for $r < a$ and for $r > a$. Make sure you specify the direction of the field in addition to the magnitude.

c) Explicitly calculate the curl of your E-field inside and outside the solenoid and comment on what you get. Is it consistent with Faraday's law in differential form?