Radiation pressure.

EM waves carry energy: $\vec{S}$ represents power flux
they also carry momentum, linear + angular.

Recall $\text{pressure} = \dfrac{\text{force}}{\text{area}} = \dfrac{\text{energy}}{\text{vol.}}$

From the stress tensor,

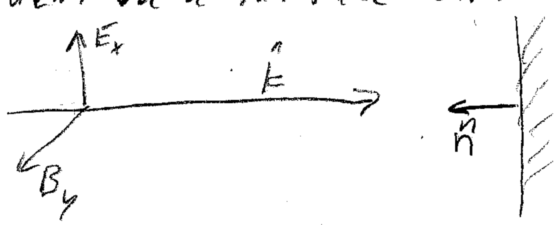
$$\vec{F} = \oint_S \overleftrightarrow{T} \cdot \hat{n}\, da \quad = \text{force on surface}$$

$\hat{n}$ points out of surface

so pressure is force directed into the surface
(shear stress is along surface)

$$P = -\hat{n} \cdot \overleftrightarrow{T} \cdot \hat{n}$$

Linear polarized plane wave $\vec{k} = k_0 \hat{z}$ $\quad \vec{E} = E_x \hat{x}$
incident on a surface with $\hat{n} = -\hat{z}$ $\quad \vec{B} = B_y \hat{y}$



wave is absorbed

since coord system is aligned with $\vec{E}, \vec{B}$, $\overleftrightarrow{T}$ is diagonal

$$\overleftrightarrow{T} = \frac{1}{8\pi} \begin{pmatrix} +E^2 - B^2 & & \\ & -E^2 + B^2 & \\ & & -E^2 - B^2 \end{pmatrix}$$

+ goes on axis of field

$$\overleftrightarrow{T} \cdot \hat{n} = \frac{E^2 + B^2}{8\pi} \hat{z}$$

$$P = -\hat{n} \cdot \overleftrightarrow{T} \cdot \hat{n} = \frac{E^2 + B^2}{8\pi} \quad \text{(instantaneous)}$$

pressure = energy density

time average:

$$\to \langle P \rangle = \frac{\frac{1}{2}E_0^2 + \frac{1}{2}B_0^2}{8\pi} = E_0^2/8\pi$$

Alternative 2

$$\vec{F} = \frac{d}{dt}\vec{p} \qquad \text{force = rate of change of momentum}$$

we found before that the complete force on charges + currents is

$$\vec{F} = \underbrace{\oint_S \overleftrightarrow{T}\cdot\hat{n}\,da - \frac{1}{c^2}\frac{\partial}{\partial t}\int \vec{S}\,d\tau}_{\frac{d}{dt}\vec{P}_{Em}} = \frac{d}{dt}\vec{P}_{matter}$$

rearrange to state

$$\oint \overleftrightarrow{T}\cdot\hat{n}\,da = \frac{d}{dt}\vec{P}_{Em} + \frac{d}{dt}\vec{P}_{matter}$$

this is a statement of conservation of momentum.
in terms of densities:

$$\nabla\cdot\overleftrightarrow{T} - \frac{\partial}{\partial t}\vec{g}_{Em} = \frac{\partial}{\partial t}\vec{g}_{matter} \qquad \text{like continuity eqn.}$$

mom dens. of field is

$$\vec{g}_{Em} = \frac{1}{4\pi c}(\vec{E}\times\vec{B}) = \frac{1}{c^2}\vec{S}$$

$$P = \frac{F}{A} \sim \langle \frac{\partial}{\partial t}\int \frac{1}{c^2}\vec{S}\,dz \rangle = \langle -\frac{i\omega}{ik}\frac{1}{c^2}S \rangle = \frac{\langle S \rangle}{c}$$

$$= \frac{E_0^2}{8\pi} \quad \text{since } \langle S \rangle = c\langle \mathcal{E} \rangle$$

beam momentum goes along $\vec{k}$ which is along ray

Alt 3 photon picture:

$$\mathcal{E}_{photon} = \hbar\omega$$
$$p_{phot} = \hbar k = \hbar\omega/c$$

beam intensity $= I = \mathcal{E}_{tot}/(\text{area}\cdot\text{time})$
$= N\hbar\omega/($ $)$
$=$ photon flux $\cdot\, \hbar\omega$
$\mathcal{F}\cdot\hbar\omega$

$\therefore$ pressure $= I/c$
$= \mathcal{F}\hbar k$

Angular momentum:

classically $\vec{L} = \vec{r}\times\vec{p}$

ang mom density $\rightarrow$ $\vec{r}\times\dfrac{\vec{S}}{c^2}$

connects to R, L circ. polarization states, Plane waves in conductors.

A natural extension of the Maxwell equations to wave prop. is to include the conductivity

- Ohms Law $\vec{J} = \sigma \vec{E}$ implies resistance + loss

→ anticipate attenuation of a wave.

one form: $e^{ik_0 z} \cdot e^{-\alpha z}$ → complex k, or complex refr. index

Caution: as ω becomes large $\sigma \to \sigma(\omega)$

- cannot use DC values of σ at optical frequencies.
- in this case it's much better to treat ε as complex.

New term in Maxwell:

$$\nabla \times B - \frac{\varepsilon \mu}{c} \frac{\partial \vec{E}}{\partial t} = \frac{4\pi}{c} \mu \vec{J}_f = \frac{4\pi \sigma \mu}{c} \vec{E}$$

↳ from $\nabla \times H \to \nabla \times B/\mu$

→ wave equation:

$$\nabla^2 \vec{E} - \frac{4\pi \sigma \mu}{c^2} \frac{\partial \vec{E}}{\partial t} - \frac{\varepsilon \mu}{c^2} \frac{\partial^2 E}{\partial t^2} = 0$$

what about ρ_f?

From continuity $\nabla \cdot \vec{J}_f = -\frac{\partial \rho_f}{\partial t}$

Ohms Law → $\frac{\partial \rho_f}{\partial t} = -\sigma \nabla \cdot \vec{E} = -4\pi \frac{\sigma}{\varepsilon} \rho_f$

solution $\rho(t) = \rho_0 e^{-4\pi\sigma \frac{t}{\varepsilon}}$ $\tau = \varepsilon / 4\pi\sigma$

for large σ, ρ_f dissipates quickly

- require $\omega \ll 4\pi\sigma/\varepsilon$
- but σ is based on collision time ($\sim 1/\nu_{coll}$)

Use σ model for "quasistatic" EM $\omega \lesssim RF$

High frequencies:

1) $\sigma(\omega)$
2) $\varepsilon \to \tilde{\varepsilon}(\omega) = \varepsilon_R + i\varepsilon_I$, $n(\omega) = n_R + i n_I$
3) Keep J_f, ρ_f and use a model for collisions
   - used in plasma physics

Dimensions of σ

in SI: $\varepsilon = \varepsilon_{rel}\varepsilon_0$ $\mu = \mu_{rel}\mu_0$

Gaussian $\varepsilon = \varepsilon_{rel}$ $\mu = \mu_{rel}$

conductivity: $\vec{J} = \sigma\vec{E}$ same in both.

in SI, wave eqn is

$$\nabla^2\vec{E} - \sigma\mu\,\partial_t\vec{E} - \varepsilon\mu\,\partial_t^2 E = 0$$

compare terms: SI $\varepsilon\mu = \varepsilon_{rel}\mu_{rel}(\varepsilon_0\mu_0) = \frac{\varepsilon_{rel}\mu_{rel}}{c^2}$

SI $\sigma\mu = \sigma\mu_{rel}\mu_0$ $(\sigma)^{SI} \sim \Omega^{-1}m^{-1}$

Gauss $\frac{4\pi\sigma\mu}{c^2}$ $(\sigma)^{gauss} \sim s^{-1}$

$$(\sigma)^{gauss} = \frac{(\sigma)^{SI}\mu_0 c^2}{4\pi} = \frac{\sigma^{SI}}{4\pi\varepsilon_0}$$

for Cu $\sigma = 59.6\times10^6/\Omega m$

$\to 5.2\times10^{17}\,s^{-1}$ in Gaussian.

$4\pi\sigma/\varepsilon \sim 10^{18}$ rad/s but ε, σ depend on freq. even at lower freq.

soft x-ray.

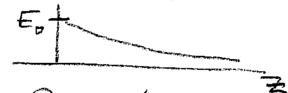
Damped waves

We want to extend our treatment of plane waves to the case where the wave amplitude changes with propagation.

Attenuation usually takes the form of an exponential decay

e.g. radioactive decay $\quad \frac{dN}{dt} = -\alpha N \rightarrow N(t) = N_0 e^{-\alpha t}$

in wave eqn: $\quad \frac{d^2E}{dz^2} = +\alpha^2 E \quad \rightarrow E(z) = E_0 e^{\pm \alpha z}$



note + sign $\rightarrow$ damped or growing exp. solns.

$-$ sign $\rightarrow$ oscillatory waves.

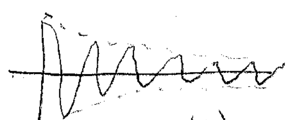
General case: complex k

$$\frac{d^2E}{dz^2} = -\tilde{k}^2 E \quad \rightarrow E(z) = E_0 e^{\pm i(k_R + i k_I)z}$$

with $\tilde{k} = k_R + i k_I$

now our wave is (w/ + sign)

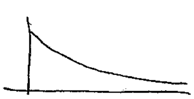
$$E_0 \, e^{i k_R z} \, e^{-k_I z}$$



Note similarity to damped SHO (same math)

What is physical interpretation?

we will find two cases

1) $\tilde{k}$ is pure imaginary $\rightarrow$  no oscillations

here, no power lost in this region

wave is reflected (entirely) examples: total int. reflect. cutoff in waveguide

2) $\tilde{k}$ is complex $\rightarrow$ damped, propagating wave.

here some or all power is absorbed.

examples: waves in metals, absorbing dielectrics.

Wave propagation

$$\nabla^2 \vec{E} - \frac{4\pi\sigma\mu}{c^2} \partial_t \vec{E} - \frac{\varepsilon\mu}{c^2} \partial_t^2 \vec{E} = 0$$

let $E \sim e^{-i\omega t}$ $\quad \partial_t \to -i\omega$

$$\nabla^2 \vec{E} + \left(i \frac{4\pi\sigma}{c^2}\mu\omega + \varepsilon\mu \frac{\omega^2}{c^2} \right) \vec{E} = 0$$

$$\nabla^2 \vec{E} = -\left(\varepsilon\mu + i \frac{4\pi\sigma\mu}{\omega} \right) \frac{\omega^2}{c^2} \vec{E}$$

this has solutions $e^{i\vec{k}\cdot\vec{r}}$ with $\vec{k} = \tilde{k}\hat{k}$ complex.

dispersion relation:

$$\tilde{k}^2 = \left(\varepsilon + i\frac{4\pi\sigma}{\omega} \right) \mu \frac{\omega^2}{c^2}$$

options:

1) complex dielectric $\quad \tilde{\varepsilon} = \varepsilon_R + i\varepsilon_I$

$\varepsilon_R = \varepsilon \quad \varepsilon_I = 4\pi\sigma/\omega$

2) complex index $\quad \tilde{n}^2 \equiv \tilde{\varepsilon}$

$\tilde{n} \equiv n_R + i n_I$ (literature: $n + ik$)

$\tilde{n}^2$ (not $|\tilde{n}|^2$)

$$= \underbrace{n_R^2 - n_I^2}_{\varepsilon_R} + i \underbrace{2 n_R n_I}_{\varepsilon_I}$$

don't try $\sqrt{\tilde{n}^2} = \tilde{\varepsilon}$ too hard!

3) book: $\quad \hat{k} = \alpha + i\beta \quad$ I use $\tilde{k}$ so that $\hat{k}$ is unit vector.

will also see $\tilde{k} = k + i\kappa$