MATH 225 – Worksheet #3 Name: Solutions

Sec:_____

To get full credit, you must show all of your work.

1. Develop a spreadsheet to solve the following problems using Euler's formula. Make sure to have the headers: k, t, y, and f(t,y) written into your printout. Attach your printouts to this worksheet.

   a. Approximate the IVP: $\frac{dy}{dt} = e^{2t} + y - t$, y(0) = 0 with step size of .2 on the interval $0 \le t \le 2$. See Attached

   b. Now approximate the same IVP on the same interval with a step size of .1 instead. See Attached.

   c. The exact solution to this IVP is: $y(t) = e^{2t} + t + 1 - 2e^{t}$. At the endpoint, $t = 2$, which is more accurate part (a) or part (b)?

$y(2) = e^4 + 2 + 1 - 2e^2 \approx 42.82$

part (a): $y(2) \approx 29.98$, Error $= |42.82 - 29.98| \approx 12.84$

part (b): $y(2) \approx 35.70$, Error $= |42.82 - 35.70| \approx 7.12$

$\boxed{\text{(b) is more accurate}}$

2. Given $\frac{dy}{dt} = y(y+3)(y-5)$,

   a. Sketch the phase line and classify all of the equilibrium points.

Equil. Solutions:

$\frac{dy}{dt} = y(y+3)(y-5) = 0$

$y = 0,\ y = -3,$

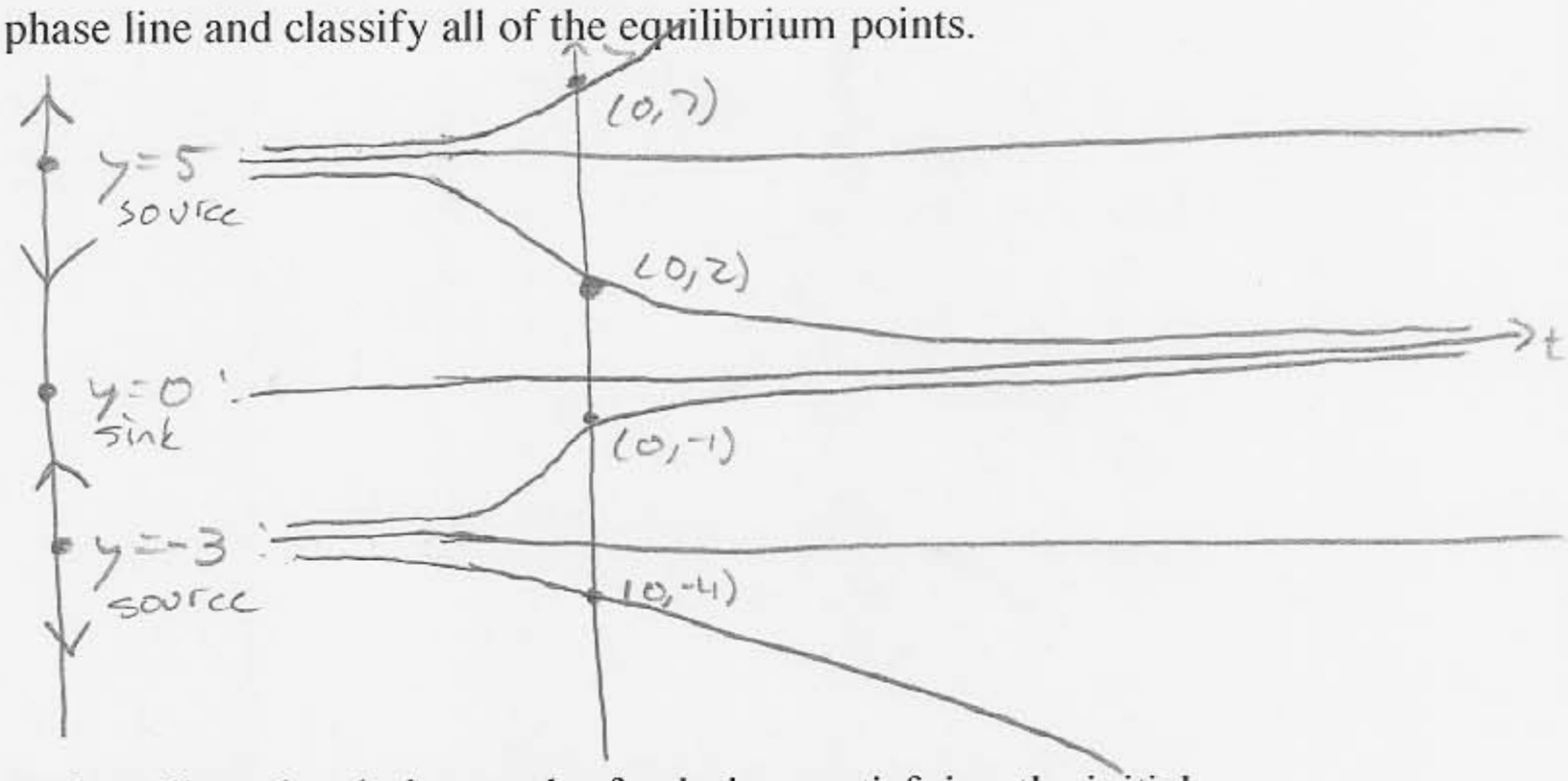
$y = 5$

$\left.\frac{dy}{dt}\right|_{y=1} = 1(4)(-4) < 0$

$\left.\frac{dy}{dt}\right|_{y=6}: 6(9)(1) > 0$

$\left.\frac{dy}{dt}\right|_{y=-1}: -1(2)(-6) > 0$

$\left.\frac{dy}{dt}\right|_{y=-4}: -4(-1)(-9) < 0$



   b. Next to your phase line, sketch the graph of solutions satisfying the initial conditions: $y(0) = -4, y(0) = -1, y(0) = 2, y(0) = 7$. Put your part (b) on one pair of axis.

   c. Describe the long-term behavior, for all of t, of the solution that satisfies the initial condition y(0) = 2.

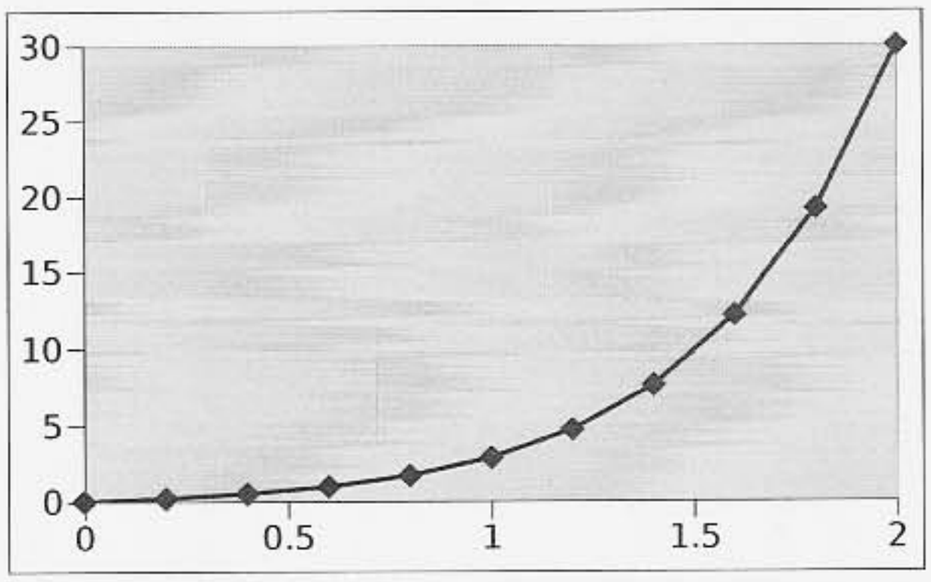
$t \to \infty,\ y \to 0$

$t \to -\infty,\ y \to 5$

1.(a)

EULER'S METHOD

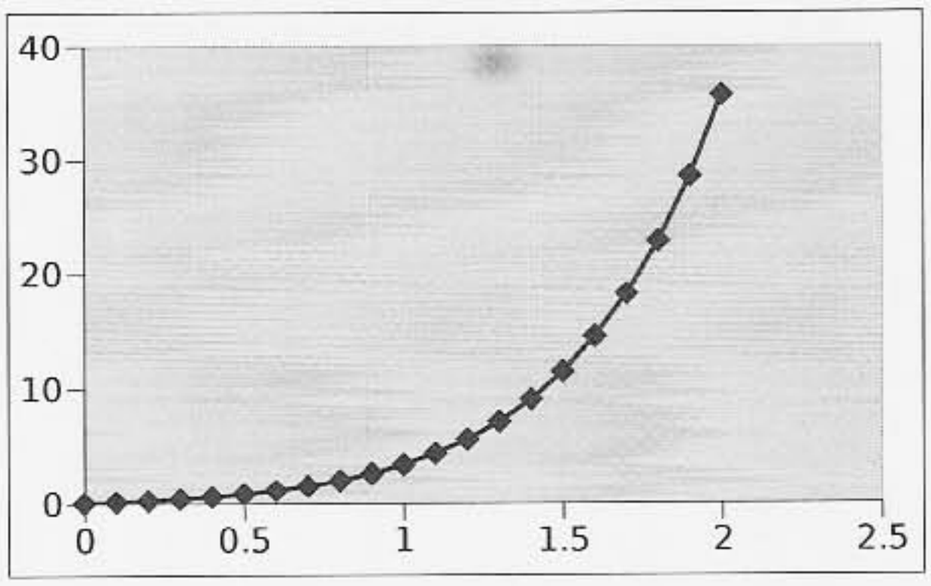
k	t_k	y_k	m_k=f(t_k,y_k)	delta t
0	0	0.0000000000	1.0000000000	0.2
1	0.2	0.2000000000	1.4918246976	0.2
2	0.4	0.4983649395	2.3239058680	0.2
3	0.6	0.9631461131	3.6832630359	0.2
4	0.8	1.6997987203	5.8528311447	0.2
5	1	2.8703649492	9.2594210482	0.2
6	1.2	4.7222491589	14.5454255395	0.2
7	1.4	7.6313342668	22.6759810379	0.2
8	1.6	12.1665304744	35.0990606715	0.2
9	1.8	19.1863426087	53.9845770523	0.2
10	2	29.9832580191	82.5814080523	0.2



1.(b)

EULER'S METHOD

k	t_k	y_k	m_k=f(t_k,y_k)	delta t
0	0	0.0000000000	1.0000000000	0.1
1	0.1	0.1000000000	1.2214027582	0.1
2	0.2	0.2221402758	1.5139649735	0.1
3	0.3	0.3735367732	1.8956555736	0.1
4	0.4	0.5631023305	2.3886432590	0.1
5	0.5	0.8019666564	3.0202484849	0.1
6	0.6	1.1039915049	3.8241084276	0.1
7	0.7	1.4864023477	4.8416023145	0.1
8	0.8	1.9705625791	6.1235950035	0.1
9	0.9	2.5829220795	7.7325695439	0.1
10	1	3.3561790339	9.7452351328	0.1
11	1.1	4.3307025471	12.2557160466	0.1
12	1.2	5.5562741518	15.3794505324	0.1
13	1.3	7.0942192050	19.2579572400	0.1
14	1.4	9.0200149290	24.0646617001	0.1
15	1.5	11.4264810991	30.0120180222	0.1
16	1.6	14.4276829013	37.3602130984	0.1
17	1.7	18.1637042111	46.4278042585	0.1
18	1.8	22.8064846370	57.6047190807	0.1
19	1.9	28.5669565450	71.3681410383	0.1
20	2	35.7037706489	88.3019206820	0.1



3. Given $\frac{dy}{dt} = \sin(y^2)$,

a. Sketch the phase line and classify all of the equilibrium points on the interval $[-\pi, \pi]$.

Equil. Solns

$\sin(y^2) = 0$

$y^2 = n\pi$, $n = 0, +1, +2, \ldots$ ($y^2 \neq -\pi, \ldots$ $y \neq \pm\sqrt{-\pi} \Rightarrow$ imaginary number)

$y = \pm\sqrt{n\pi}$

In $[-\pi, \pi]$: $y = 0$, $y = \sqrt{\pi}$, $y = -\sqrt{\pi}$, $y = \sqrt{2\pi}$, $y = -\sqrt{2\pi}$, $y = \sqrt{3\pi}$, $y = -\sqrt{3\pi}$

Linearization Thm: $f(y) = \sin(y^2)$

$f'(y) = 2y\cos(y^2)$

$f'(0) = 0$: No help

$f'(\sqrt{\pi}) = 2\sqrt{\pi}(-1) < 0$: sink

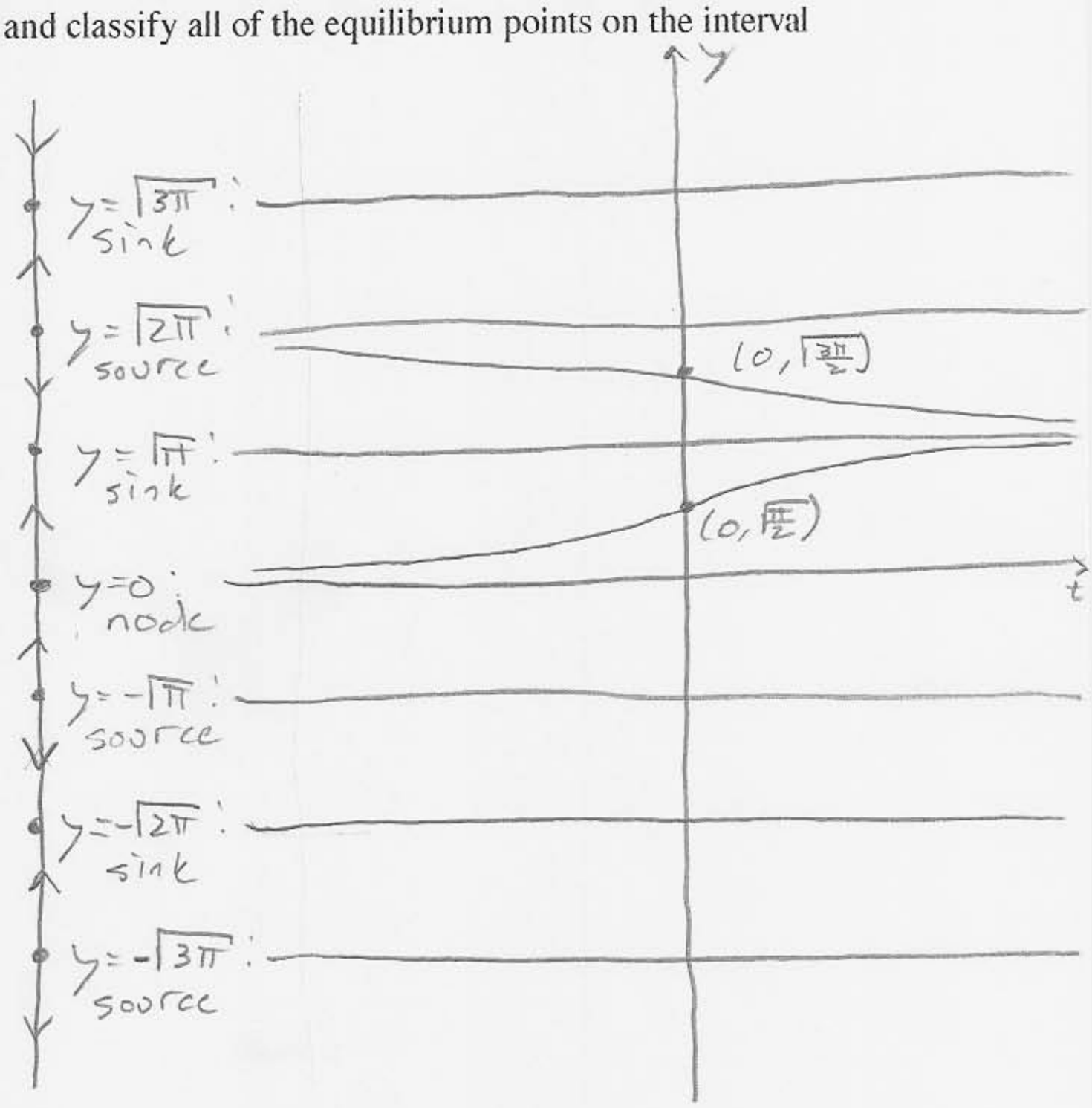
$f'(\sqrt{2\pi}) = 2\sqrt{2\pi}(1) > 0$: source

$f'(\sqrt{3\pi}) = 2\sqrt{3\pi}(-1) < 0$: sink

$f'(-\sqrt{\pi}) = -2\sqrt{\pi}(-1) > 0$: source

$f'(-\sqrt{2\pi}) = -2\sqrt{2\pi}(1) < 0$: sink

$f'(-\sqrt{3\pi}) = -2\sqrt{3\pi}(-1) > 0$: source



b. Next to your phase line, sketch the graph of solutions satisfying the initial conditions: $y(0) = \sqrt{\frac{\pi}{2}}, y(0) = \sqrt{\frac{3\pi}{2}}$. Put your part (b) on one pair of axis.

c. Describe the long-term behavior, for all of t, of the solution that satisfies the initial condition $y(0) = \sqrt{\frac{\pi}{2}}$.

$t \to \infty$, $y \to \sqrt{\pi}$

$t \to -\infty$, $y \to 0$

d. What theorem guarantees that the solutions with different initial conditions will not cross in the graph from part (b)?

Existence & Uniqueness Theorem (or Uniqueness Theorem)

4. Find the solution to the following differential equations or initial-value problems using the Method of Undetermined Coefficients.

a. $\frac{dy}{dt} = 2y + 5e^{-t}$

$\frac{dy}{dt} - 2y = 5e^{-t}$

y_h: $\frac{dy_h}{dt} - 2y_h = 0$

$y_h = e^{rt}$, $y_h' = re^{rt}$

$(r-2) = 0$

$r = 2$, $y_h = Ke^{2t}$

y_p: $\frac{dy_p}{dt} - 2y_p = 5e^{-t}$

$y_p = \alpha e^{-t}$, $y_p' = -\alpha e^{-t}$

$-\alpha e^{-t} - 2\alpha e^{-t} = 5e^{-t}$

$-3\alpha = 5$

$\alpha = -\frac{5}{3}$

$y_p = -\frac{5}{3}e^{-t}$

$\boxed{y(t) = y_h + y_p = Ke^{2t} - \frac{5}{3}e^{-t}}$

b. $\frac{dy}{dt} + 2y = 5e^{-2t}$

y_h: $\frac{dy_h}{dt} + 2y_h = 0$

$y_h = e^{rt}$, $y_h' = re^{rt}$

$r + 2 = 0$

$r = -2$

$y_h = Ke^{-2t}$

y_p: $\frac{dy_p}{dt} + 2y_p = 5e^{-2t}$

$y_p = \alpha e^{-2t}$, $y_p' = -2\alpha e^{-2t}$

$\cancel{-2\alpha e^{-2t} + 2\alpha e^{-2t} = 5e^{-2t}}$

$\cancel{0 = 5e^{-2t}}$

2nd try: $y_p = \alpha te^{-2t}$

$y_p' = \alpha e^{-2t} - 2\alpha te^{-2t}$

$\alpha e^{-2t} - \cancel{2\alpha te^{-2t}} + \cancel{2\alpha te^{-2t}} = 5e^{-2t}$

$\alpha = 5$

$y_p = 5te^{-2t}$

$\boxed{y(t) = Ke^{-2t} + 5te^{-2t}}$

c. $\frac{dy}{dt} + y = 3\sin(2t), y(0) = 1$

y_h: $\frac{dy_h}{dt} + y_h = 0$

$y_h = e^{rt}$ $y_h' = re^{rt}$

$r + 1 = 0$

$r = -1$

$y_h = Ke^{-t}$

y_p: $\frac{dy_p}{dt} + y_p = 3\sin(2t)$

$y_p = A\sin(2t) + B\cos(2t)$

$y_p' = 2A\cos(2t) - 2B\sin(2t)$

$2A\cos(2t) - 2B\sin(2t) + A\sin(2t) + B\cos(2t) = 3\sin(2t)$

$(2A+B)\cos(2t) + (-2B+A)\sin(2t) = 0\cos(2t) + 3\sin(2t)$

$2A + B = 0$ $\quad -2B + A = 3$

$B = -2A$, $\quad 4A + A = 3$, $A = \frac{3}{5}$

$B = -\frac{6}{5}$

$y_p = \frac{3}{5}\sin(2t) - \frac{6}{5}\cos(2t)$

$\boxed{y(t) = Ke^{-t} + \frac{3}{5}\sin(2t) - \frac{6}{5}\cos(2t)}$

$y(0) = K + 0 - \frac{6}{5} = 1$

$K = \frac{11}{5}$

$\boxed{y(t) = \frac{11}{5}e^{-t} + \frac{3}{5}\sin(2t) - \frac{6}{5}\cos(2t)}$